Give Peace a Chance: The Effect of Ownership and Asymmetric Information on Peace*

Luis C. Corchón

Departamento de Economía, Universidad Carlos III, Madrid, Spain.

Anıl Yıldızparlak †

Departamento de Economía, Universidad Carlos III, Madrid, Spain.

Abstract

We study the possibility of peace when two countries fight a war over the ownership of a resource. War is always the outcome of the game played by rational countries -under complete or asymmetric information- when there is no pre-established distribution of the resource among countries. When there is such a distribution of the resource, under complete information peace is feasible for some initial distributions of the resource, whereas under asymmetric information there are two classes of equilibria: *Peaceful Equilibria*, in which peace has a positive probability, and *Aggressive Equilibria*, which assign probability one to war. Surprisingly, a little asymmetric information may yield war.

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†Corresponding author. *Tel*: +34916249367. *E-mail address*: ayildizp@eco.uc3m.es *Postal Address*: Universidad Carlos III de Madrid, Calle Madrid 126, 28903, Getafe, Madrid, Spain.

1 Introduction

Rationalist theories explain war as the rational choice of countries (see Hirshleifer (1991), Skaperdas (2002) and surveys by Garfinkel and Skaperdas (2007) and Jackson and Morelli (2011)). This approach shows how factors such as trade, long-term relationships, political bias and the distribution of resources amplify or efface the incentives for war (see Skaperdas and Syropoulos (1996), (2001), Garfinkel and Skaperdas (2000), Jackson and Morelli (2007) and Beviá and Corchón (2010)).[1]

In this paper, we consider a conflict arising between two countries for the control of a resource. Our emphasis will be on the effects of asymmetric information and the distribution of the resource prior to the conflict. To address the second issue, we consider two setups: the Undistributed Resource Game (UR), where countries have no prior ownership of the resource, and the Fully Distributed Resource Game (FDR), where there is a pre-existing distribution of the resource. Examples of the first situation are the Scramble for Africa between all major European powers in 1881-1914 and the Great Game played by British and Russian empires in 1813-1907 for the control of Afghanistan. With respect to the second setup, the distribution of the resource may be achieved by an agreement (such as the treaty of Tordesillas, 1494, in which Spain and Portugal divided South America according to a suggestion made by the Pope), for cultural reasons (language, history), geographical features (a river, a strait, a mountain chain) or by a previous conflict as in the case of Cyprus.[2]

Other than the initial position, UR and FDR are identical two-stage games. In the first stage, countries decide if they declare war or not. If one of the countries declares war, war occurs in the second stage. If both countries decide not to fight, there is peace and they get zero payoff in UR, and their prior distribution of the resource in FDR.

[1]A forerunner of this approach is Clausewitz (1832, Chapter 1), who noted that "War is akin to a card game".

[2]For a recent application of mediation to war, see Horner, Morelli and Squintani (2011).

We first study complete information which serves us as a benchmark case. For UR, war is the only equilibrium outcome. The explanation is that since peace yields the status quo, i.e., zero payoff outcome, a rational country always prefers conflict. For FDR, we show that there is a set of divisions of the resource such that, in equilibrium, both countries will choose peace. The reason is that the status quo for each country is her share of the resource. The possibility of losing this share makes countries reluctant to go to war.

Next, we consider asymmetric information. We assume that country one has private information on how valuable the resource is for her and may have a high or a low valuation (type) of the resource at stake. Country two has only a prior probability that country one is of the high or low type. As an illustration, think that country one has done research on the existence of a valuable resource in a territory under dispute and the results of this research are in the hands of this country only. Also, a country might be uncertain about the willingness of the other country to fight. However, by observing the declaration of her rival, country two has the possibility of inferring the type she faces at war.

We prove for UR that war is the unique Perfect Bayesian equilibrium outcome of the asymmetric information game. For FDR, we find that there are two classes of equilibria. The first class, Peaceful Equilibria, contains equilibria that assigns at least a positive probability to peace. In the second class, all equilibria assign probability one to war.

In the Peaceful Equilibria, there are two kinds of equilibria. In the first, the high type declares war and the low type is peaceful while country two is also peaceful. Consequently, when country one declares war, country two infers she is fighting a high type and subsequently countries play under complete information. There are distributions of the resource for which this equilibrium exists except when the valuations of country two and the low type are low. The reason for the lack of existence is due to the ability of the low type to fake a high type when country two is very weak, because this country will be very insufficiently armed in a conflict. If the low type's valuation is high enough, one can find a distribution of the resource to sustain peace as country two would demand a very low share of the resource.

In the second class of equilibrium, every type and country choose peace. This equilibrium does not exist when all the following possibilities occur jointly:

1. There is a low probability that country one has a high valuation.
2. There is a large dispersion in the possible valuations of country one.
3. The strength and/or valuation of country two is high.

Point 1 is counterintuitive. It says peace can not be achieved when we are close to complete information! The interpretation is that the share of country one is dictated by its high valuation, but when there is a high probability that country one is weak, war looks like a good prospect for country two, especially when the likely low type has a low valuation of the resource (point 2) and country two is powerful or values the resource a lot (point 3). Note that, despite the fact that the high type is unlikely, war occurs with probability one.

We end this section by reviewing the literature. Schelling (1980) and Fearon (1995) suggested that asymmetric information is a possible cause of war. An early model of war including asymmetric information is by Brito and Intriligator (1985). A thorough discussion of the effects of incomplete information on war is in Jackson and Morelli (2011, p. 10). They conclude that "If the cost of war is low enough, then country B is better off simply going to war and taking its chances rather than reaching such an unfavorable bargain." Our findings complete this intuition by showing a list of the causes of war and by highlighting the role of the initial share in the resource. In particular, our results show that *relative* magnitudes matter, namely the dispersion of valuations in country one and the relative strength of country two, and that a low probability that country one has a high valuation is also bad for peace.

The rest of the paper goes as follows. Section 2 spells out the model. Section 3 studies the full information case. Section 4 considers asymmetric information.

Finally, Section 5 presents our final comments.

2 The Model

Two countries dispute a divisible resource which they value in V_1 and V_2, respectively. In case of war, they incur sunk expenses of g_1 and g_2. Let p_i be the probability that i obtains the resource after the war.[3] p_i is determined by an asymmetric contest success function of the following form:

$$p_i = \begin{cases} \frac{\beta_i g_i}{\sum_{j=1}^{2} \beta_j g_j} & \text{if } g_1 + g_2 > 0 \\ \frac{\beta_i}{\beta_i + \beta_j} & \text{if } g_1 + g_2 = 0 \end{cases} \tag{1}$$

where $\beta_i \in (0, \infty)$ is the productivity of country i in war efforts.[4] Defining $\theta \equiv \beta_2/\beta_1$ as the relative productivity of country two in war, the contest success function when $g_1 + g_2 > 0$ can be rewritten as:

$$p_1 = \frac{g_1}{g_1 + \theta g_2} \quad \text{and} \quad p_2 = \frac{\theta g_2}{g_1 + \theta g_2} \tag{2}$$

Both countries are risk-neutral. In case of war, their expected payoffs are

$$u_1 = V_1 \frac{g_1}{g_1 + \theta g_2} - g_1 \quad \text{and} \quad u_2 = V_2 \frac{\theta g_2}{g_1 + \theta g_2} - g_2, \tag{3}$$

The game is played as follows: In the first stage, countries decide to declare war or not. If a country declares war, we will say that this country makes the necessary preparations for war, even though a formal declaration might not be issued. The decisions at this stage are perfectly observed when the next stage begins. In the second stage, if one of the countries declared war, the conflict is waged and payoffs are delivered. Otherwise, we have peace.

[3] p_i may also be equivalently interpreted as the share of the resource obtained, but for concreteness, we will follow the probabilistic interpretation throughout the paper.

[4] In Appendix 6.1, we deal with a more general contest success function and show that our results under complete information do not change qualitatively.

Our two-stage model of war avoids the problem arising in one-stage models where starting war is a dominant strategy. The shortcomings of our approach are that we do not allow for surprise attacks and that, once war is prepared, there is no way of achieving peace. We discuss both issues in turn.

The most famous historical surprise attacks -the attacks of Nazi Germany on Russia and Japan on the United States, both in December 1941- ended with the attackers ultimately being defeated. The surprise attack of Japan on Russia in February 1905 was more successful but produced only minimal casualties. In these examples, the fate of war was determined by battles fought later on.[5] Therefore, it seems that, at least in these historical examples, surprise attacks do not decisively influence the outcome of the conflict. This observation agrees with our model, where the outcome of war relies entirely on the contest success function (2) and the expenses made later on, but surprise attacks may play a role in our model in terms of starting a war (see below).

In many actual wars, there was a decision that led inevitably to a conflict: raising an army in 16^{th}-17^{th} century Europe, with no means of supporting itself except by plunder, or accumulating a large number of troops on the border or even a surprise attack which -as we argued before- has no consequence on the outcome of war. It is true that the final spark in some actual wars was somehow random, like in the Spanish-American war of 1898 or the First World War in 1914. However, it may be argued that the decisions made by these countries in previous years made war inevitable sooner or later. Thus our assumption that once war is declared there is no turning back can be regarded as a simplification of a more complex situation involving random elements but captures that certain actions, that are irrelevant to the outcome of the conflict, make peace impossible.

We consider two setups. They differ in the status quo prior to the game. In the *Undistributed Resource Game* (henceforth *UR*), countries own nothing out of the resource in dispute. The early race to the conquest of South America

[5] The decisive battles in these wars were Kursk (July-August 1943), Midway (June 1942) and Tsushima (May 1905). See Beevor (2012).

between Spain and Portugal at the end of 15^{th} century and so-called *Scramble for Africa* would be examples of this kind of game.[6] In the *Fully Distributed Resource Game* (henceforth *FDR*), there is a pre-existent full distribution of the resource which defines particular shares for each contestant. In this case, countries engage in war for the full resource, but the payoffs of peace are the shares of the resource given by the initial distribution. The Napoleonic Wars and World Wars I and II are examples of this kind of game.

3 Full Information Case

Assume that all the parameters defining the game are common knowledge between the two countries. We solve the game beginning with the second stage. Assuming that there is war, first order conditions (henceforth FOC) of expected payoff maximization for each country are:

$$\frac{\partial u_1}{\partial g_1} = V_1 \frac{\theta g_2}{(g_1 + \theta g_2)^2} - 1 = 0 = V_2 \frac{\theta g_1}{(g_1 + \theta g_2)^2} - 1 = \frac{\partial u_2}{\partial g_2} \tag{4}$$

We verify in Appendix 6.1 that these conditions are sufficient. Thus, war efforts are given by (4), which implies that

$$\frac{V_2}{V_1} = \frac{g_2}{g_1}. \tag{5}$$

Substituting (5) into (4) and defining $\gamma \equiv \frac{V_2}{V_1}$, we obtain the full information war effort, g_i^F, for $i = 1, 2$.

$$g_1^F = \frac{V_1 \theta \gamma}{(1 + \theta \gamma)^2} \tag{6}$$

$$g_2^F = \frac{V_2 \theta \gamma}{(1 + \theta \gamma)^2} \tag{7}$$

[6] It can be argued that both historical examples did not end up in war, but by reinterpreting war efforts as the cost of colonization and the resulting p_i's as the shares in South America/Africa, our analysis is still applicable.

Substituting expressions (6) and (7) into (3), we obtain the equilibrium payoff for each country:

$$u_1^F = \frac{V_1}{(1+\theta\gamma)^2} \tag{8}$$

$$u_2^F = \frac{V_2\theta^2\gamma^2}{(1+\theta\gamma)^2} \tag{9}$$

The payoffs above are strictly positive. Thus, we have the following result:

Proposition 1 *Under complete information, war is the unique equilibrium outcome of UR.*

Let us now study FDR. Denote the pre-established distribution of the resource in the hands of country one as $\varepsilon \in (0,1)$. As the resource is fully distributed, the share of country two is $1-\varepsilon$. For peace to hold in equilibrium, the following condition should hold for the first country.

$$u_1^F = V_1 \frac{1}{(1+\theta\gamma)^2} \leq \varepsilon V_1. \tag{10}$$

which amounts to

$$\frac{1}{(1+\theta\gamma)^2} \leq \varepsilon \tag{11}$$

Performing a similar calculation for the second country, we obtain

$$\varepsilon \leq \frac{1+2\theta\gamma}{(1+\theta\gamma)^2}. \tag{12}$$

Then, if peace is achieved, conditions (11) and (12) imply

$$\frac{1}{(1+\theta\gamma)^2} \leq \varepsilon \leq \frac{1+2\theta\gamma}{(1+\theta\gamma)^2} \tag{13}$$

Note that the right-hand side (henceforth, RHS) of identity (13) is always greater than the left-hand side (henceforth, LHS) of it. Moreover, both sides are positive and less than 1. Thus, we have proved the following:

Proposition 2 *Under complete information, there is a set of divisions of the* *resource given by (13), where peace is the unique equilibrium outcome of FDR.*

Equation (13) defines the set of shares yielding peace as an equilibrium. Let $x = \theta\gamma$. x measures the magnitude of asymmetries in parties rooted in the fighting power and/or valuation of the resource. As player 2 gets stronger in valuation/fighting power, then $x \to \infty$. And if player 1 gets stronger in valuation/ fighting power, then $x \to 0$.

The length of the set for which peace holds as an equilibrium is given by

$$\Phi(x) = \frac{2x}{(1+x)^2}.$$

$\Phi(x) \to 0$ as $x \to 0$, or $x \to \infty$ and it has a maximum at $x = 1$, i.e., when players are identical. Thus asymmetries between contestants make it harder to find a division that achieves peace as an equilibrium outcome. This is because a stronger contestant would demand a larger share when she has a large possibility of a victory in a decisive battle.

4 Asymmetric Information

Under asymmetric information, UR and FDR are two-stage games with observed actions and incomplete information where players (countries) simultaneously choose an action at each stage. The types correspond to different possible valuations of the resource. Let $\mathbb{V}_i$ be the type set for country i, $i = 1, 2$. We assume that country one has two possible types denoted by V_H and V_L with $V_H > V_L$. Country two has only one type denoted by V_2. Now let us introduce the following pieces of notation

$$\gamma = \frac{V_2}{V_H},\ \gamma \in (0,1);\ \ \rho = \frac{V_L}{V_H},\ \rho \in (0,1)$$

Note that V_2/V_L equals γ/ρ. Let us assume that $V_H > V_2 > V_L$ which implies:

$$1 > \gamma > \rho. \tag{14}$$

Condition (14) excludes cases in which country two is "very weak" or "very strong" in terms of valuations of the resource and is introduced mainly, for analytic convenience. Next, we introduce an assumption which guarantees that war expenses are non-negative:

$$\frac{\sqrt{\rho}}{1-\sqrt{\rho}} \geq \theta\gamma = x \tag{15}$$

The LHS of (15) is increasing in ρ. The RHS of (15) measures the magnitude of asymmetries between countries rooted in the fighting power and/or valuation of the resource between country two and the high type country one (recall that $\theta\gamma$ was denoted by x in the previous section). Thus (15) says that the difference between the possible valuations of country one is large compared to the asymmetries between players.

Country two has prior beliefs that country one is of the high type with probability $\pi \in [0,1]$, and that she is of low type with probability $1-\pi$.

In the first stage, countries choose an action $a_i \in A = \{P, D\}$ where P is to stay peaceful and D is to declare war. These actions are revealed at the end of this stage. When player one is of type V_H (resp. V_L), her war effort is denoted by g_H (resp. g_L).

At the end of the first stage, player two updates her beliefs about the type of player one, based on the history of actions available in the first stage, denoted by h. We denote $\mathcal{H}$ as the set of all possible histories, i.e., $\mathcal{H} = \{DD, DP, PD, PP\}$.

The posterior belief of player two about the type of player one is $\mu : \mathcal{H} \to \Delta(\mathbb{V}_1)$, where $\Delta(\mathbb{V}_1)$ is the set of all probability distributions on $\mathbb{V}_1$. With a slight abuse of notation, we denote the posterior beliefs as $(\mu(V_H|h), \mu(V_L|h)) = (\mu, 1-\mu)$. Therefore, country two ex-post believes that country one is of the high type with probability $\mu \in [0,1]$, and that she is of the low type with probability $1-\mu$. As $\mu \leq 1$, from (15) it follows that

$$\sqrt{\rho} \geq \mu x \left[1 - \sqrt{\rho}\right] \tag{16}$$

Let $s = (a, g)$ be a strategy profile consisting of the choice of an action in the first stage and the choice of effort in the second stage. For FDR, we define the following payoff functions for players one and two.

$$u_j\left(s|h,\mu,V_j\right) = \begin{cases} V_j \frac{g_j}{g_j+\theta g_2} - g_j & \text{if } h \neq (P,P) \\ \varepsilon V_j & \text{if } h = (P,P) \end{cases} \quad j = H, L \tag{17}$$

$$u_2\left(s|h,\mu,V_2\right) = \begin{cases} V_2 \left(\mu \frac{\theta g_2}{g_H+\theta g_2} + (1-\mu)\frac{\theta g_2}{g_L+\theta g_2}\right) - g_2 & \text{if } h \neq (P,P) \\ (1-\varepsilon) V_2 & \text{if } h = (P,P) \end{cases} \tag{18}$$

For UR, the payoffs concerning $h = (P,P)$ are replaced by zero. Otherwise, payoffs are identical to those in (17) and (18).

We define a Perfect Bayesian Equilibrium (henceforth, PBE) for UR and FDR following Fudenberg and Tirole (1991). For the sake of simplicity, we only consider pure strategies throughout our analysis.

Definition 3 *For UR and FDR, a PBE in pure strategies is a strategy profile $s^* = (a^*, g^*)$ and posterior beliefs μ such that:*

(H) $\forall i = H, L, 2$, $\forall h \in \mathcal{H}$, and given $\mu \in [0,1]$;

$$u_i\left(a^*, g^*|h,\mu,V_i\right) \geq u_i\left(a^*, g_i, g^*_{-i}|h,\mu,V_i\right)$$

(P) $\forall i = H, L, 2$ and given $\pi \in [0,1]$,

$$u_i\left(a^*, g^*|\pi,V_i\right) \geq u_i\left(a_i, a^*_{-i}, g^*|\pi,V_i\right)$$

(B1) Denote $\sigma^*(a_1|h, V_1)$ as a degenerate probability distribution over A.

$$\mu' = \frac{\pi\sigma^*(a_1|h, V_H)}{\pi\sigma^*(a_1|h, V_H) + (1-\pi)\sigma^*(a_1|h, V_L)} \text{ and}$$

$$\pi\sigma^*(a_1|h, V_H) + (1-\pi)\sigma^*(a_1|h, V_L) > 0$$

(B2) $\forall h$ and $\forall a_2, \hat{a}_2 \in A$, $\mu = \mu(V_H|h = (a_1, a_2)) = \mu(V_H|h = (a_1, \hat{a}_2))$.

(H) and (P) impose each continuation strategy to be a Bayesian equilibrium of the game starting with history h and beliefs corresponding to that history. (H) is the requirement concerning second-stage histories and beliefs, i.e., h, and μ, whereas (P) is the requirement concerning war declaration. (B1) states that the posterior beliefs, given the history in stage two, ought to obey the Bayes' rule whenever possible. We impose that σ^* must be a degenerate probability distribution as we only consider pure strategies. Lastly, (B2) says that, given the signal sent by player one, the posterior beliefs of player two are independent of her action in the first stage.

Throughout this section, we consider separating and pooling equilibria. A separating (resp. pooling) equilibrium is a strategy profile in which different types of player one choose different (resp. the same) actions in the first stage.

We start by analyzing UR. Let us assume for a moment that we have war in the second stage. Then, the expected payoffs of each player given by (17) and (18), for $h \neq (P, P)$, are strictly concave. Thus FOCs guarantee a unique maximum. For the time being, we disregard non-negativity constraints. Using the reaction functions derived in Appendix 6.3, we get the equilibrium war effort of each country and each player as:

$$g_H^* = \sqrt{\rho} V_H \frac{\left[\frac{\rho}{x} + (1-\mu)\left(1-\sqrt{\rho}\right)\right]\left[1-\mu\left(1-\sqrt{\rho}\right)\right]}{\left[\frac{\rho}{x} + 1 - \mu(1-\rho)\right]^2} \tag{19}$$

$$g_L^* = \rho V_H \frac{\left[\frac{\rho}{x} - \mu\sqrt{\rho}\left(1-\sqrt{\rho}\right)\right]\left[1-\mu\left(1-\sqrt{\rho}\right)\right]}{\left[\frac{\rho}{x} + 1 - \mu(1-\rho)\right]^2} \tag{20}$$

$$g_2^* = \frac{\rho V_H}{\theta}\left[\frac{1-\mu\left(1-\sqrt{\rho}\right)}{\frac{\rho}{x}+1-\mu\left(1-\rho\right)}\right]^2 \tag{21}$$

Note that g_H^* and g_2^* are both strictly positive and by (16), g_L^* also is strictly positive. Then, substituting (19)-(21) into (17) and (18), we get the following equilibrium payoffs for each player and type.

$$u_H^* = V_H\left[\frac{\frac{\rho}{x}+\left(1-\mu\right)\left(1-\sqrt{\rho}\right)}{\frac{\rho}{x}+1-\mu\left(1-\rho\right)}\right]^2 \tag{22}$$

$$u_L^* = \rho V_H\left[\frac{\frac{\rho}{x}-\mu\sqrt{\rho}\left(1-\sqrt{\rho}\right)}{\frac{\rho}{x}+1-\mu\left(1-\rho\right)}\right]^2 \tag{23}$$

$$u_2^* = V_2\frac{\left[1-\mu\left(1-\rho\right)\right]\left[1-\mu\left(1-\sqrt{\rho}\right)\right]^2}{\left[\frac{\rho}{x}+1-\mu\left(1-\rho\right)\right]^2} \tag{24}$$

Note that all payoffs above are strictly positive.

Lemma 4 *There is no PBE for UR in which player two is peaceful and at least one type of player one is peaceful.*

Proof. Assume first $a = (P, P, P)$. By requirement (H), $g = (g_H^*, g_L^*, g_2^*)$ are given by (19)-(21) for any system of beliefs, which in turn implies $u_i^* > 0$, $\forall i$. Denote $\hat{h}$ as the alternative history in which player two chooses D. Then, using (B2), if player two deviates to D, she obtains $u_2^* > 0$.

Now assume that $a = (D, P, P)$. In this profile, $u_2\left(a^*, g^*|h, v_2, \mu\right) = \pi u_2^*$. Using (B2), player two gets u_2^* if she deviates, which is larger. The proof concerning the profile $a^* = (P, D, P)$ is identical to the one above. ■

Lemma 5 *There is no separating equilibrium in which player two declares war.*

Proof. Assume $a = (D, P, D)$ or $a = (P, D, D)$. Take the former profile, and assume it is part of the PBE. The consistent beliefs are $\mu = 1$, and there is war with probability one. The payoff of the high type is given by substituting $\mu = 1$ into (22), whereas the deviation payoff is given by substituting $\mu = 0$ into (22).

Thus, for a high type player not to deviate:

$$\left[\frac{\frac{\rho}{x}}{\frac{\rho}{x}+\rho}\right]^2 > \left[\frac{\frac{\rho}{x}+1-\sqrt{\rho}}{\frac{\rho}{x}+1}\right]^2 \iff \frac{\frac{\rho}{x}+\rho}{\frac{\rho}{x}+1} > \sqrt{\rho} \tag{25}$$

The payoff of the low type implied by the profile is given by substituting $\mu = 0$ into (23), whereas the deviation payoff is given by substituting $\mu = 1$ into (23). Therefore, for a low type player not to deviate:

$$\left[\frac{\frac{\rho}{x}}{\frac{\rho}{x}+1}\right]^2 \geq \left[\frac{\frac{\rho}{x}+\rho-\sqrt{\rho}}{\frac{\rho}{x}+\rho}\right]^2 \iff \frac{\frac{\rho}{x}+\rho}{\frac{\rho}{x}+1} \leq \sqrt{\rho} \tag{26}$$

which contradicts (25). The proof concerning $a = (P, D, D)$ is identical. ■

Proposition 6 *War is the unique PBE outcome of UR.*

Proof. By lemmata 4 and 5, there is not any peaceful PBE. Thus, if we conclude that at least one of the two remaining profiles, (D, D, P) or (D, D, D), is an equilibrium, we prove the proposition.

Now assume that $a = (D, D, P)$. War occurs. By (B2), player two is indifferent between the two actions, whereas if any type deviates, players achieve the zero payoff outcome. As war always pays off a positive expected amount, no type will have any incentive to deviate. ■

Proposition 6 says that the results obtained in UR under asymmetric information parallel the ones from UR under complete information. Specifically, peace is impossible in this setting under complete or asymmetric information.

We now study FDR under asymmetric information. First, we show that there is a unique separating equilibrium. In particular, by lemmata 7 and 8 below, we first eliminate some of the candidate equilibrium profiles. Then, in proposition 9, we confirm that the only remaining strategy profile is a PBE.

Lemma 7 *There is no separating equilibrium for FDR in which player two declares war.*

Proof. Assume $a = (D, P, D)$ or $a = (P, D, D)$. War occurs. Thus, the conditions are the same as (25) and (26) and the proof concerning FDR is the same as the proof concerning UR. ■

Lemma 8 *There is no separating equilibrium for FDR under asymmetric information in which the high type is peaceful, and the low type declares war when player two is peaceful.*

Proof. $a = (P, D, P)$. The consistent beliefs imply $\mu = 0$. Substituting $\mu = 0$ into equations (22) and (23), we find the equilibrium payoffs of war for the high type and the low type, respectively, as follows:

$$V_H \left[\frac{\frac{\rho}{x} + 1 - \sqrt{\rho}}{\frac{\rho}{x} + 1} \right] \quad \text{and} \quad V_L \left[\frac{\frac{\rho}{x}}{\frac{\rho}{x} + 1} \right]$$

For this profile to be an equilibrium, the high type should be better off under peace:

$$\varepsilon \geq \left[\frac{\frac{\rho}{x} + 1 - \sqrt{\rho}}{\frac{\rho}{x} + 1} \right]$$

Moreover, the low type has to be better off declaring war

$$\varepsilon \leq \left[\frac{\frac{\rho}{x}}{\frac{\rho}{x} + 1} \right]$$

which constitutes a clear contradiction. ■

Proposition 9 *There is a unique separating equilibrium of FDR in which country 2 and the low type are peaceful as the high type declares war if and only if:*

$$\left[1 - \frac{x}{\sqrt{\rho}\,(1 + x)} \right]^2 \leq \varepsilon \leq \frac{\rho\,(\rho + 2x)}{(\rho + x)^2} \tag{27}$$

Proof. Assume $a = (D, P, P)$ and condition (15) is satisfied. The consistent beliefs imply $\mu = 1$. Then, condition (H) implies the following ex-post payoffs for the high type and player two, respectively:

$$u_H = \frac{V_H}{(1+x)^2} \text{ and } u_2 = \frac{V_2 x^2}{(1+x)^2}$$

If player one is a low type, peace is obtained with the following ex-post payoffs.

$$u_L = \varepsilon \rho V_H,\ u_2 = (1-\varepsilon) V_2$$

For a to be a part of the PBE, a high type ought to choose peace:

$$u_H \geq u_H^D = \varepsilon V_H \iff \frac{1}{(1+x)^2} \geq \varepsilon \tag{28}$$

Also, a low type player ought not to deviate to declaring war. Given a, player two will act as if she were facing a high type. Substituting $\mu = 1$ into (23), we get:

$$u_L^D = \rho V_H \left[\frac{\frac{\rho}{x} - \sqrt{\rho}\left(1-\sqrt{\rho}\right)}{\frac{\rho}{x} + \rho} \right]^2$$

The condition for the low type player is $u_L = \varepsilon \rho V_H \geq u_L^D$, which reduces to

$$\left[\frac{\frac{\rho}{x} - \sqrt{\rho}\left(1-\sqrt{\rho}\right)}{\frac{\rho}{x} + \rho} \right]^2 = \left[1 - \frac{x}{\sqrt{\rho}\,(1+x)} \right]^2 \leq \varepsilon \tag{29}$$

Finally, player two should not deviate. The expected payoff of player two is:

$$u_2 = \pi \frac{V_2 x^2}{(1+x)^2} + (1-\pi) V_2 (1-\varepsilon)$$

whereas the deviation payoff given the belief $\mu = 1$ is:

$$u_2^D = \pi \frac{V_2 x^2}{(1+x)^2} + (1-\pi) \frac{V_2 (x/\rho)^2}{(1+(x/\rho))^2}$$

Hence, the condition for player two is $u_2 \geq u_2^D$, implying:

$$\frac{1+2(x/\rho)}{(1+(x/\rho))^2} = \frac{\rho(\rho+2x)}{(\rho+x)^2} \geq \varepsilon \tag{30}$$

Combining (28)-(30), the condition for a being a part of the PBE is:

$$\left[1-\frac{x}{\sqrt{\rho}\,(1+x)}\right]^2 \leq \varepsilon \leq \min\left\{\frac{\rho\,(\rho+2x)}{(\rho+x)^2},\frac{1}{(1+x)^2}\right\}$$

Note that the LHS of the condition above is less than the second term in the RHS of it. Hence, we drop the condition concerning the high type, implying that the sufficient condition for the existence of the equilibrium is given by (27). By lemmata 7 and 8, there is no other separating equilibrium. Combining conditions (28)-(30), we get the result. ■

Now we look for pooling equilibria. As mentioned before, in a pooling equilibrium, both types of player one either declare war or stay peaceful.

Proposition 10 *For FDR, given beliefs $\mu\,(V_H|h)=\pi$ where $h\in\{(D,.)\,,(P,.)\}$ and g satisfies (H);*

(i) $a=(D,D,D)$ is a part of the pooling equilibrium if off-the-path beliefs $\mu\,(V_H|\,(P,.))=\mu^o$ satisfy:

$$\pi\leq\mu^o \text{ if } x\geq\sqrt{\rho}\text{ , and } \pi>\mu^o \text{ if } x<\sqrt{\rho} \tag{31}$$

(ii) $a=(P,P,D)$ is a part of the pooling equilibrium if off-the-path beliefs $\mu\,(V_H|\,(D,.))=\mu^o$ satisfy:

$$\begin{gathered}\pi\leq\mu^o \text{ if } x\geq\sqrt{\rho}\text{ , } \pi>\mu^o \text{ if } x<\sqrt{\rho}\\ \text{and}\\ 1-\varepsilon\leq\frac{\left[1-\pi\,(1-\rho)\right]\left[1-\pi\left(1-\sqrt{\rho}\right)\right]^2}{\left[\frac{\rho}{x}+1-\pi\,(1-\rho)\right]^2}\end{gathered} \tag{32}$$

(iii) $a=(D,D,P)$ is a part of the pooling equilibrium if

$$\varepsilon\leq\left[\frac{\frac{\rho}{x}-\pi\sqrt{\rho}\left(1-\sqrt{\rho}\right)}{\frac{\rho}{x}+1-\pi\,(1-\rho)}\right]^2 \tag{33}$$

(iv) $a=(P,P,P)$ is a part of the pooling equilibrium if off-the-path beliefs

$\mu \left(V_H \right| (D, .)) = \mu^o$ *satisfy:*

$$\begin{aligned} \varepsilon &\geq \left[\frac{\frac{\rho}{x} + (1-\mu^o)\left(1-\sqrt{\rho}\right)}{\frac{\rho}{x} + 1 - \mu^o (1-\rho)} \right]^2 \\ &\quad \textit{and} \\ 1-\varepsilon &\geq \frac{\left[1-\pi(1-\rho)\right]\left[1-\pi\left(1-\sqrt{\rho}\right)\right]^2}{\left[\frac{\rho}{x}+1-\pi(1-\rho)\right]^2} \end{aligned} \tag{34}$$

Proof.

(i) For (D, D, D), deviation payoffs for each type are given by (22) and (23) for $\mu = \mu^o$. Then, condition (P) implies that off-the-equilibrium beliefs have to obey:

$$\frac{\frac{\rho}{x} + (1-\mu^o)\left(1-\sqrt{\rho}\right)}{\frac{\rho}{x} + 1 - \mu^o (1-\rho)} \leq \frac{\frac{\rho}{x} + (1-\pi)\left(1-\sqrt{\rho}\right)}{\frac{\rho}{x} + 1 - \pi (1-\rho)}$$

$$\frac{\frac{\rho}{x} - \mu^o\sqrt{\rho}\left(1-\sqrt{\rho}\right)}{\frac{\rho}{x} + 1 - \mu^o (1-\rho)} \leq \frac{\frac{\rho}{x} - \pi\sqrt{\rho}\left(1-\sqrt{\rho}\right)}{\frac{\rho}{x} + 1 - \pi (1-\rho)},$$

for the high and the low type, respectively. Simplifying the inequalities above, we see that they hold iff $(\pi - \mu^o)\left(\sqrt{\rho} - x\right) \geq 0$, which is condition (31).

(ii) For (P, P, D), the first two conditions are equivalent to the conditions for part (i). Given $\mu^o = \pi$ by (B2), for player two not to deviate to peace, $u_2^* \geq (1-\varepsilon) V_2$, implying the condition.

(iii) If (D, D, P) is a part of the PBE, $u_H^* \geq \varepsilon V_H$, and $u_L^* \geq \varepsilon\rho V_H$. By (22) and (23), $u_L^* \geq \varepsilon\rho V_H \;\Rightarrow\; u_H^* \geq \varepsilon V_H$ because:

$$\frac{\frac{\rho}{x} - \mu\sqrt{\rho}\left(1-\sqrt{\rho}\right)}{\frac{\rho}{x} + 1 - \mu (1-\rho)} \leq \frac{\frac{\rho}{x} + (1-\mu)\left(1-\sqrt{\rho}\right)}{\frac{\rho}{x} + 1 - \mu (1-\rho)}$$

Therefore, the binding condition is for the low type. By (B2), player two is indifferent between her two actions.

(iv) If (P, P, P) is a part of the PBE, $u_H^* \leq \varepsilon V_H$, $u_L^* \leq \varepsilon\rho V_H$, and $u_2^* \leq$

$(1-\varepsilon) V_2$. As $u_H^* \geq u_L^*$ by (22) and (23), $u_H^* \leq \varepsilon V_H \;\Rightarrow\; u_L^* \leq \varepsilon \rho V_H$ because:

$$\frac{\frac{\rho}{x} + (1-\mu^o)\left(1-\sqrt{\rho}\right)}{\frac{\rho}{x} + 1 - \mu^o (1-\rho)} \geq \frac{\frac{\rho}{x} - \mu^o \sqrt{\rho}\left(1-\sqrt{\rho}\right)}{\frac{\rho}{x} + 1 - \mu^o (1-\rho)}$$

Thus, the binding condition is for the high type. By (B2), if player two declares war, she has the same beliefs; i.e., $\mu^o = \pi$. Thus,

$$1-\varepsilon \geq \frac{\left[1-\pi(1-\rho)\right]\left[1-\pi\left(1-\sqrt{\rho}\right)\right]^2}{\left[\frac{\rho}{x} + 1 - \pi(1-\rho)\right]^2}$$

■

We now focus on two equilibria in which peace is the possible outcome: The separating equilibrium in Proposition 9, where only the high type declares war, and the pooling equilibrium in Proposition 10 part (*iv*), in which all types and all players are peaceful.

First, we consider the pooling equilibrium. Note that the pooling equilibrium implies probability one for peace. Therefore, we choose to name this equilibrium Definitely Peaceful Equilibrium (DPE). This equilibrium holds if (34) is satisfied.

We assume that off-the-equilibrium path beliefs are the prior beliefs; i.e., $\mu\left(V_H \middle| (D,.)\right) = \mu^o = \pi$. Other assumptions about those beliefs are possible, but we focus on this case for the sake of simplicity. Thus, the two conditions of (34) boil down to:

$$1 - \frac{\left[1-\pi(1-\rho)\right]\left[1-\pi\left(1-\sqrt{\rho}\right)\right]^2}{\left[\frac{\rho}{x} + 1 - \pi(1-\rho)\right]^2} \geq \varepsilon \geq \left[\frac{\frac{\rho}{x} + (1-\pi)\left(1-\sqrt{\rho}\right)}{\frac{\rho}{x} + 1 - \pi(1-\rho)}\right]^2 \tag{35}$$

Rearranging (35), we reach the following condition for peace to hold in equilibrium:

$$\frac{1}{x} \geq \frac{(1-\pi)\left(1-\sqrt{\rho}\right)^2 \left\{(1-\pi)^2 - \pi^2 \rho\right\}}{2\rho\sqrt{\rho}\left[1-\pi\left(1-\sqrt{\rho}\right)\right]} \tag{36}$$

The RHS of (36) will be denoted as $F(\pi,\rho)$. We have two cases:

If $(1-\pi)^2/\pi^2 \leq \rho$, the RHS of (36) is non positive. Therefore, there is a distribution of the resource that achieves peace. This case arises if ρ is very close to one (so both types have similar valuations) or when π is very close to one, so the probability of a high type is overwhelming. In both cases, the uncertainty is small. Note that this case is compatible with our assumption in (15) iff $\sqrt{\rho} > \max\{\frac{1-\pi}{\pi}, \frac{x}{1+x}\}$.

If $(1-\pi)^2/\pi^2 > \rho$, the RHS of (36) is positive and things are more involved. Firstly, note that this case is compatible with (15) iff $\frac{1-\pi}{\pi} \geq \sqrt{\rho} \geq \frac{x}{1+x}$ which implies that $1 \geq \pi + x + 2\pi x$; i.e., π, and x cannot be very large at the same time. Now, let us discuss the role of each parameter separately.

The role of x is clear. When the strength of country two or her valuation is low, peace holds because this country could only demand a small share and this is always feasible.

To find the role of π, we partially differentiate the right-hand side of (36) with respect to π and find the following expression:

$$sign\frac{\partial F(\pi,\rho)}{\partial \pi} = sign\left[-2+2\pi\left(1+\sqrt{\rho}\right)-\sqrt{\rho}\right]. \tag{37}$$

It is easy to see that

$$sign\frac{\partial F(\pi,\rho)}{\partial \pi} < 0 \Leftrightarrow \pi < \frac{2+\sqrt{\rho}}{2\left(1+\sqrt{\rho}\right)}$$

but the last inequality is implied by $(1-\pi)^2/\pi^2 > \rho$. Thus $F(\pi,\rho)$ decreases with π and we conclude that a large value of π gives peace a good chance. This is very intuitive because uncertainty is very small. However, a sufficiently low value of π makes war very likely. Indeed, when $\pi \simeq 0$, the condition for war is

$$2\frac{\rho\sqrt{\rho}}{\left(1-\sqrt{\rho}\right)^2} < x. \tag{38}$$

In order to show that (38) is compatible with our assumptions, we provide an example. Let $x = 2/5$ (e.g., $\theta = 1$ and $\gamma = 2/5$), and $\rho = 1/9$, so $1 > \gamma > \rho$. By (15), the low type puts a positive effort in war. However, we will have war at $\pi \to 0$, as $2\rho\sqrt{\rho}/\left(1-\sqrt{\rho}\right)^2 = 1/6$.

Note that the LHS of (38) is increasing in ρ, so in this case, war arises as a combination of a low probability of country one being of the high type, a large valuation and/or strength of country two and a small valuation for the low type country one. In all these cases, the share inducing the high type to be peaceful looks too expensive for country two, which has a good chance of winning a sizable chunk of the prize by going to war.

It is remarkable that despite the fact that when $\pi = 0$, war cannot happen in FDR (see Proposition 2), war is perfectly possible when π is very close to zero. Even more, war is more likely the closer the value of π is to zero. This reveals an interesting discontinuity in the prevention of war.

Finally, it is easily seen that $\frac{\partial F(\pi,\rho)}{\partial \rho} < 0$ for $F(\pi,\rho) \geq 0$. Again, if ρ is close to 1, $F(\pi,\cdot)$ is close to 0 and peace has a fair chance. This is because we are close to complete information. However, if ρ is low, $F(\pi,\cdot)$ could be large and make peace impossible. This is because the low type will make very little effort in war and will lose it with a high probability, but its share in the resource is determined by the high type, so war looks like a good prospect for country two.[7]

Summing up, from the discussion above, we learn that the failure of the existence of DPE for FDR is due to:

1. Large relative strength and/or valuation of country two.
2. Low probability that country one has a high valuation.
3. Large dispersion in the possible valuations of country one.

The mechanism under which war occurs is that country one appears, in expected terms, as a weak opponent in war, but peace can only be avoided when the share

[7] Even though condition (15) gets harder to be obtained as $\rho \to 0$, it can be shown that there are values of other parameters that allow condition (15) to be fulfilled.

of this country is given by the characteristics of the high type.

Now we consider the separating equilibrium in Proposition 9. Realize that in this equilibrium, war takes place with probability π. Hence we call this equilibrium Possibly Peaceful Equilibrium (PPE). The relevant condition for the existence of this equilibrium is given by:

$$\left[1 - \frac{x}{\sqrt{\rho}\,(1+x)}\right]^2 \leq \frac{\rho\,(\rho+2x)}{(\rho+x)^2} \tag{39}$$

In Appendix 6.4, we show that the inequality (39) holds when ρ and x are not very small. For example, for $\rho = 0.01$, $\gamma = 0.02$ and $\theta = 1$, (39) fails to hold. The explanation is that when x is very small, the effort of player two is biased to small values when she observes a declaration at the beginning of the second stage. However, ex-ante, because of the probability of a very weak opponent, she still demands some considerable share of the resource. Thus, if the low type deviates and declares war, she can benefit from the beliefs of player two, who believes that she fights a powerful rival.

We also show in Appendix 6.4 that if x is large, implying that player two is strong, an increase in ρ makes the possible partitions of the resource that sustain PPE larger. This result partially parallels the results obtained under complete information. Realize that in PPE, the peaceful players are both the low type and player two. Thus if their strength/valuation are similar, the possible partitions that satisfy both parties is a large set, given that the partition does not satisfy the high type. However, when x is small, an increase in ρ makes the possible partitions that allow for PPE to be smaller, as when country two is very weak and there is not much difference between a high and a low type, the corresponding partitions are a small set. Note, however, that there are possible partitions supporting PPE, when country two is very weak, because she only demands a small amount of resources, unlike the case where ρ and x are both low.

Note that if $\varepsilon \geq 1/(1+x)^2$, PPE fails to exist, as the high type would be rich to risk a war. Combined with π being sufficiently low, DPE also fails to exist, implying that in the remaining equilibria war occurs.

5 Final Comments

In this paper, we studied the role of information and resource ownership in conflicts. In order to conform with our initial motivation we only mentioned warfare throughout the paper. However, we can also apply our model to litigation. An example is divorce proceedings in which a previous partition of the resource, e.g. total wealth of the couple and full/partial custody rights, may or may not exist. This situation would be a straightforward application of FDR and UR.

We now summarize our results. In the benchmark case of complete information, when the resource is not distributed, war always occurs. When the resource is distributed, war is a consequence of the interplay between asymmetries of contestants (embedded in x) and the distribution of ownership (ε). When the latter does not reflect the former adequately, war occurs. The good news is that war can always be stopped by the appropriate distribution of resources. These conclusions agree with some theories about wars in 17^{th} century Europe when the Spanish empire was militarily weak but owned large territories and France was militarily powerful and contended part of the territories owned by Spain (a similar case occurred in the 20^{th} century with the British Empire and Germany as players). Our theory, contrary to some theories of war, stresses the role of relative asymmetries; it is not the size of the prize that triggers war but the relative distribution of it in relationship to the relative strength of the armies.[8]

Under asymmetric information, when the resource is not distributed, the outcome is always war. Thus, asymmetric information does not alter the picture when there is no prior ownership. However, when the resource is distributed, informational asymmetries make a difference regarding war or peace. We dis-

[8] "Unsustainable practices led to ....agriculturally marginal lands having to be abandoned again. Consequences for society included ... wars" Diamond (2005) p. 15.

tinguish between war in a pooling equilibrium, in which both types declare war, and war in a separating equilibrium, in which only the high type declares war.

In a pooling equilibrium, if the uninformed country assigns a low probability that the informed country has a high type, war always occurs. This is noteworthy because it shows that a little asymmetric information may cause war. Apparently inexplicable facts like the Nazi invasion of the USSR might be explained by this mechanism. A weak USSR owned “too much land” with respect to the military power the Nazis expected from her. The German beliefs of a weak USSR were justified by the “Great Purge” in which Joseph Stalin disposed of many capable military leaders, resulting in a significant weakening of the Red Army. A similar case might be made with respect to the attack of the Japanese Empire on Pearl Harbor. Japanese leaders believed that the United States had too much influence in the Pacific relative to their expected willingness to fight (which they thought was low). The beliefs of the Japanese elite were justified by the fact that in 1941 the US economy was still recovering from the Great Depression.

In a separating equilibrium only the high type declares war. This is reminiscent of the Russo-Japanese war (1904-05), where Japan declared war on Russia. The cause of the war was, for one, that Russia did not want to recognize the Japanese influence in Korea and, furthermore, the belief of many influential Russians that (despite the complete victory of Japan over China in the 1894-5 war) “Japan is not a country that can issue an ultimatum to Russia” (see Ferguson (2006, Chap. 2)). In any case, deeper research is needed on the applications of the contest model to history; see Hoffman (2012) for a recent entry on this. Our paper just provides a theoretical model and sketches some possible applications.

In order to make the model tractable, we have made a number of assumptions and left aside some questions that we discuss now.

1. We assumed that after war, no compensation is paid by the loser, but there are historical examples in which compensations were paid, i.e., the Franco-

Prussian War (1870-1), World War I (1914-1918), etc. Farmer and Pecorino (1999) have shown that when the loser has to pay the expenses of the winner, total expenses might skyrocket because the winner pays nothing. It would be interesting to know how war may arise in this case. Given that payoffs under war are smaller than under no compensation, intuition suggests that peace can be even more likely in this case than under no compensation.

2. Another extension would be to consider a political bias as in Jackson and Morelli (2007). In this paper, the agent running a country might receive high profits from the victory but pay only a fraction of the cost of war. In this case, if the bias is sufficiently large, peace cannot hold.

3. Finally, bargaining for the distribution of the resource usually takes several rounds. It would be interesting to model the distributions of the resource that could be achieved by bargaining as in the model of Rubinstein (1982).

We hope that our paper sheds light on the powers and limitations of achieving peace by means of the distribution of resources and pinpoints the cases in which achieving peace by this means is bound to fail and other measures like direct UN intervention have to be taken.

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6 Appendix

6.1 Existence of SPNE with Generalized Tullock CSF

Assume the CSF is given as follows:

$$p_1 = \frac{g_1^s}{g_1^s + \theta g_2^s} \quad \text{and} \quad p_2 = \frac{\theta g_2^s}{g_1^s + \theta g_2^s}. \tag{40}$$

Then, the payoff functions of war for each player are:

$$u_1 = V_1 \frac{g_1^s}{g_1^s + \theta g_2^s} - g_1 \quad \text{and} \quad u_2 = V_2 \frac{\theta g_2^s}{g_1^s + \theta g_2^s} - g_2. \tag{41}$$

The FOCs of the payoff maximization are:

$$\frac{\partial u_1}{\partial g_1} = \frac{\partial u_2}{\partial g_2} = V_1 \frac{s\theta g_1^{s-1} g_2^s}{(g_1^s + \theta g_2^s)^2} - 1 = V_2 \frac{s\theta g_2^{s-1} g_1^s}{(g_1^s + \theta g_2^s)^2} - 1 = 0 \tag{42}$$

Solving system (42), we obtain:

$$g_1^* = V_1 \frac{s\theta\gamma^s}{(1 + \theta\gamma^s)^2} \quad \text{and} \quad g_2^* = V_2 \frac{s\theta\gamma^s}{(1 + \theta\gamma^s)^2} \tag{43}$$

These efforts are not necessarily an NE, because the Second Order Condition (SOC) of payoff maximization might not hold, so an extra argument is necessary. Consider player one. Her payoff function with $g_2 = g_2^*$ is continuous, so there is a maximum of this function over the interval $[0, V_1]$. Even though the maximization on the definition of an NE is on the real line, in equilibrium, no rational player will spend more on war than her valuation. Thus, the aforementioned maximum can be either located at the extremes, i.e. $g_1^* = 0$, $g_1^* = V_1$ or in interior, in which case this maximum is g_1^*. Payoffs for the first two options are zero and negative, respectively, so if we show that payoffs for player one are non-negative when evaluated at g_1^* and g_2^*, then g_1^* is a best reply to g_2^*. The same argument applies for player two. Letting $V_2 = \gamma V_1$, we need:

$$V_1 \frac{1 + \theta\gamma^s - s\theta\gamma^s}{(1 + \theta\gamma^s)^2} \geq 0 \text{ and } V_2 \frac{\theta\gamma^s + \theta^2 (\gamma^s)^2 - s\theta\gamma^s}{(1 + \theta\gamma^s)^2} \geq 0 \tag{44}$$

or

$$1 + \theta\gamma^s \geq s\theta\gamma^s \quad \text{and} \quad \theta + \theta^2\gamma^s \geq s\theta \tag{45}$$

which when $\theta = 1$ boil down to the conditions in Nti (1999). Note that when $s \leq 1$, (45) always holds. Finally, when both players are identical ($\theta = \gamma = 1$), (45) reads that $s \leq 2$.

Some remarks are in order. First, in the NE constructed above,

$$\frac{g_2^*}{g_1^*} = \frac{V_2}{V_1} = \gamma \tag{46}$$

Thus, as it happens when $s \leq 1$, war productivity does not affect the ratio of equilibrium expenses.

Second, if (45) does not hold for at least one player, the second stage of UR will fail to have an NE in pure strategies. Even though there is an equilibrium in mixed strategies, given the difficulty of interpreting mixed strategies in our framework, we do not pursue this matter.

Finally, using (45), we can show that the conditions for peace to hold in equilibrium are

$$\frac{1 + (1 - s)\,\theta\gamma^s}{(1 + \theta\gamma^s)^2} V_1 \leq \varepsilon V_1 \quad \text{and} \quad \frac{\theta (\gamma^s)^2 + (1 - s)\,\theta\gamma^s}{(1 + \theta\gamma^s)^2} V_2 \leq (1 - \varepsilon)\, V_2$$

The conditions above boil down to the following set of divisions admitting peace.

$$\frac{1 + (1 - s)\,\theta\gamma^s}{(1 + \theta\gamma^s)^2} \leq \varepsilon \leq \frac{1 + (1 + s)\,\theta\gamma^s}{(1 + \theta\gamma^s)^2}$$

which for $s = 1$ are identical to (13).

6.2 Resource Constraints under Complete Information

Assume players one and two, respectively, own resources $R_1 = R$ and $R_2 = aR$ to be used in war, exclusively, where $a \geq 0$ is the relative amount of resources of player two. Assume $R \geq g_1$, $aR \geq g_2$, so no player can spend more than she initially owns. We now analyze both UR and FDR under this new assumption.

If player i is constrained, the marginal utility of effort is greater than the marginal cost of it at $g_i = R_i$. Formally

$$\left.\frac{\partial u_i}{\partial g_i}\right|_{g_i=R_i} = V_i \frac{\theta g_j}{(g_1 + \theta g_2)^2} - 1 \geq 0 \tag{47}$$

If (47) holds, the equilibrium effort of player i is $g_i^* = R_i$. Given the concavity of the payoff function, this condition is necessary and sufficient.

First, consider UR. There are four cases. In the first one, no player is resource-constrained, which is the case we analyzed in the main text. In the second one, both players are resource-constrained, which implies equilibrium efforts are $g_1^* = R$, and $g_2^* = aR$. Using (47), this occurs if and only if

$$V_1/R \geq (1 + a\theta)^2 / a\theta \;\; \text{and} \;\; V_2/R \geq (1 + a\theta)^2 / \theta \tag{48}$$

Note that (48) implies that, in an NE, payoffs are strictly positive. In UR, the payoff of peace is zero; hence, war is the only equilibrium outcome when both countries are constrained. Clearly, this result extends to the cases where only one player is constrained.

Now consider FDR. If peace holds in equilibrium, both countries are better off choosing peace:

$$V_1 \frac{1}{1 + a\theta} - R \leq \varepsilon V_1 \;\; \text{and} \;\; V_2 \frac{a\theta}{1 + a\theta} - aR \leq (1 - \varepsilon) V_2 \tag{49}$$

or

$$\frac{1}{1+a\theta} - \frac{R}{V_1} \leq \varepsilon \leq 1 + \frac{aR}{V_2} - \frac{a\theta}{1+a\theta} \tag{50}$$

For peace to hold, the LHS of (50) has to be less than 1 and the RHS of (50) has to be greater than 0, as $\varepsilon \in (0,1)$. These conditions are implied by (48). Clearly, the RHS of (50) has to be larger than the LHS of it. Suppose on the contrary that

$$1 + \frac{aR}{V_2} - \frac{a\theta}{1+a\theta} \leq \frac{1}{1+a\theta} - \frac{R}{V_1} \Leftrightarrow \frac{aR}{V_2} + \frac{R}{V_1} \leq 0 \tag{51}$$

which is impossible. Thus, there is an ε such that (50) holds and peace is the only equilibrium outcome. Now assume that only one player, say player one, is constrained. Thus, $g_1^* = R_1$. The best reply of player two is

$$g_2^* = \frac{\sqrt{\theta V_2 g_1} - g_1}{\theta} = \frac{\sqrt{\theta R V_2} - R}{\theta} \tag{52}$$

For peace to hold, the distribution of the resource $\varepsilon \in (0,1)$ should satisfy:

$$V_1 \sqrt{\frac{R}{\theta V_2}} - R \leq \varepsilon V_1 \quad \text{and} \quad V_2 - 2\sqrt{\frac{R V_2}{\theta}} + \frac{R}{\theta} \leq (1-\varepsilon) V_2 \tag{53}$$

Using simple algebra, we reduce (53) to the following.

$$\sqrt{\frac{R}{\theta V_2}} - \frac{R}{V_1} \leq \varepsilon \leq 2\sqrt{\frac{R}{\theta V_2}} - \frac{R}{\theta V_2} \tag{54}$$

The LHS (54) has to be less than 1. Assume otherwise. Note at g_1^* and g_2^* (47) reads:

$$\frac{V_1}{\sqrt{\theta R V_2}} \left(1 - \sqrt{\frac{R}{\theta V_2}}\right) \geq 1$$

However, the LHS being larger than 1 contradicts the condition above. The necessity of the RHS to be larger than 0 is implied again by condition (47).

The last condition is the non-emptiness of the set defined by (54). Suppose this interval is empty. Then:

$$\sqrt{\frac{R}{\theta V_2}} - \frac{R}{\theta V_2} + \frac{R}{V_1} \leq 0 \Leftrightarrow \sqrt{\frac{RV_2}{\theta}} - \frac{R}{\theta} \leq -\frac{RV_2}{V_1} \tag{55}$$

whereas by the FOC of payoff maximization of player one, we obtain that

$$\sqrt{\frac{RV_2}{\theta}} - \frac{R}{\theta} \geq \frac{RV_2}{V_1} \tag{56}$$

which together with (55) implies a contradiction. Thus, there is always a division of the resource such that peace is the unique equilibrium outcome. The case when country 2 is constrained and country 1 is not is equivalent. Summing up, proposition 2 can be generalized to include resource constraints.

6.3 Equilibrium in FDR with asymmetric information

Neglecting the non-negativity constraints, the FOCs of (17) for a high and a low type are, respectively:

$$\frac{\partial u_1}{\partial g_H} = V_H \frac{\theta g_2}{(g_H + \theta g_2)^2} - 1 = \frac{\partial u_1}{\partial g_L} = \rho V_H \frac{\theta g_2}{(g_L + \theta g_2)^2} - 1 = 0$$

Using the expressions above, we get the following best response functions for the high and the low type, respectively, as:

$$g_H = \sqrt{V_H \theta g_2} - \theta g_2 \quad \text{and} \quad g_L = \sqrt{\rho V_H \theta g_2} - \theta g_2$$

The FOC of (18) for country two is:

$$\frac{\partial u_2}{\partial g_2} = V_2 \theta \left[\mu \frac{g_H}{(g_H + \theta g_2)^2} + (1 - \mu) \frac{g_L}{(g_L + \theta g_2)^2} \right] - 1 = 0$$

Equilibrium war efforts (19)-(21) are found by substituting the best response functions of the two possible types into the FOC of the problem of country two:

$$\frac{\partial u_2}{\partial g_2} = V_2\theta \left[\mu \frac{\sqrt{V_H \theta g_2} - \theta g_2}{V_H \theta g_2} + (1-\mu) \frac{\sqrt{\rho V_H \theta g_2} - \theta g_2}{\rho V_H \theta g_2} \right] - 1 = 0$$

Using the notation $x = \theta\gamma$, we solve the equation above for g_2, and we find the equilibrium war effort of country two as:

$$g_2^* = \frac{\rho V_H}{\theta} \left[\frac{1 - \mu\left(1 - \sqrt{\rho}\right)}{\frac{\rho}{x} + 1 - \mu\left(1 - \rho\right)} \right]^2$$

Substituting the result above into the best response functions of the high and the low type, respectively:

$$g_H^* = \sqrt{\rho} V_H \left[\frac{1 - \mu\left(1 - \sqrt{\rho}\right)}{\frac{\rho}{x} + 1 - \mu\left(1 - \rho\right)} \right] - \rho V_H \left[\frac{1 - \mu\left(1 - \sqrt{\rho}\right)}{\frac{\rho}{x} + 1 - \mu\left(1 - \rho\right)} \right]^2$$

$$g_L^* = \rho V_H \frac{1 - \mu\left(1 - \sqrt{\rho}\right)}{\frac{\rho}{x} + 1 - \pi\left(1 - \rho\right)} - \rho V_H \left[\frac{1 - \mu\left(1 - \sqrt{\rho}\right)}{\frac{\rho}{x} + 1 - \mu\left(1 - \rho\right)} \right]^2$$

Simplifying the expressions above, we find the following equilibrium war efforts of the high and the low type:

$$g_H^* = \sqrt{\rho} V_H \frac{\left[\frac{\rho}{x} + (1-\mu)\left(1 - \sqrt{\rho}\right)\right]\left[1 - \mu\left(1 - \sqrt{\rho}\right)\right]}{\left[\frac{\rho}{x} + 1 - \mu\left(1 - \rho\right)\right]^2}$$

$$g_L^* = \rho V_H \frac{\left[\frac{\rho}{x} - \mu\sqrt{\rho}\left(1 - \sqrt{\rho}\right)\right]\left[1 - \mu\left(1 - \sqrt{\rho}\right)\right]}{\left[\frac{\rho}{x} + 1 - \mu\left(1 - \rho\right)\right]^2}$$

6.4 Separating Equilibrium for FDR

First we show, whenever $\rho \geq .184$ or $x \geq .28$, condition (39) holds. Notice if $\rho = .184$ and $x = .28$, assumption (15) holds, and (39) is equivalent to:

$$G\left(\rho, x\right) = \left[1 - \frac{x}{\sqrt{\rho}\left(1 + x\right)}\right]^2 - \frac{\rho\left(\rho + 2x\right)}{\left(\rho + x\right)^2} \leq 0 \tag{57}$$

We see that $G(\rho, x) \leq 0$ iff $H(\rho, x) \leq 0$, where the latter is defined as:

$$H(\rho, x) = (\rho + x)\left((1 + x)\sqrt{\rho} - x\right) - \rho(1 + x)\sqrt{\rho + 2x} \leq 0 \tag{58}$$

Note that $H(\rho, 0) = 0$, so if $\frac{\partial H(\rho,x)}{\partial x} < 0$, (39) holds. Differentiating (58):

$$\sqrt{\rho} - (\rho + 2x)(1 - \sqrt{\rho}) - \frac{\rho(1 + 3x + \rho)}{\sqrt{\rho + 2x}} \tag{59}$$

The second and the third term in (59) are negative. Let us first disregard the second term. If $\frac{\partial H(\rho,x)}{\partial x} > 0$, we have

$$\rho^3 + 2\rho^2 + x\left(6\rho + 6\rho^2 - 2\right) + x^2(9\rho) < 0 \tag{60}$$

When $x = 0$ or $x \to \infty$, (60) is impossible, so if (60) holds, there is an x for which the LHS of (60) is zero. Solving the equation:

$$x = \frac{2 - 6\rho - 6\rho^2 \pm 2\sqrt{3\rho^2 - 6\rho + 1}}{2(\rho^3 + 2\rho^2)} \tag{61}$$

If $3\rho^2 - 6\rho + 1 < 0$ - which is true for $\rho \gtrapprox .184$ - the expression under the root is negative so no solution for x exists.

Next, disregarding the third term in (59) and assuming $\frac{\partial H(\rho,x)}{\partial x} > 0$, we obtain:

$$\frac{\sqrt{\rho} - \rho + \rho\sqrt{\rho}}{2\left(1 - \sqrt{\rho}\right)} > x \tag{62}$$

If the inequality in (62) is reversed, which is possible as $\frac{\sqrt{\rho}}{1-\sqrt{\rho}} > \frac{\sqrt{\rho}-\rho+\rho\sqrt{\rho}}{2\left(1-\sqrt{\rho}\right)}$, we arrive at a contradiction. Since the LHS of (62) is increasing in ρ and for $\rho \gtrapprox .184$ we already proved that inequality (39) holds, it is sufficient that x is larger than the LHS of (62) evaluated at $\rho \gtrapprox .184$ and this yields $x \gtrapprox 0.28$.

Now let us partially differentiate $G(\rho, x)$ wrt ρ:

$$
\begin{aligned}
\frac{\partial G\left(.,x\right)}{\partial\rho} &= \frac{2x^{2}}{\left(\rho+x\right)^{3}}-\frac{x}{\left(1+x\right)\rho^{3/2}}+\frac{x^{2}}{\rho^{2}\left(1+x\right)^{2}} \\
&\geq \frac{2x^{2}}{\left(1+x\right)^{3}}-\frac{x}{\left(1+x\right)\rho^{3/2}}+\frac{x^{2}}{\left(1+x\right)^{2}} \\
&= \frac{x}{1+x}\left(\frac{2x}{\left(1+x\right)^{2}}-\frac{1}{\rho^{3/2}}+\frac{x}{1+x}\right)
\end{aligned}
$$

So $\frac{2x}{(1+x)^2} - \frac{1}{\rho^{3/2}} + \frac{x}{1+x} > 0 \Rightarrow \frac{\partial G}{\partial \rho} > 0$. This inequality is equivalent to

$$
\rho > \left(\frac{\left(1+x\right)^{2}}{3x+x^{2}}\right)^{\frac{2}{3}} \tag{63}
$$

The RHS of (63) is less than one for $x > 1$, so if ρ is sufficiently close to one, (63) holds. Thus, when the uninformed country is powerful and there is very little uncertainty about the valuation of the informed country, an increase in ρ makes the interval larger, given π.

Now, we manipulate $\frac{\partial G(.,x)}{\partial \rho}$ to get:

$$
\frac{\partial G\left(.,x\right)}{\partial\rho} = x\left(\frac{2x}{(\rho+x)^{3}}-\frac{1}{(1+x)\rho^{3/2}}+\frac{x}{\rho^{2}(1+x)^{2}})\right)
$$

As $x \to 0$, the expression in brackets is negative, so $\frac{\partial G}{\partial \rho} < 0$. Therefore, when the uninformed country is very weak, an increase in ρ makes the interval smaller, given π.