Online Appendix

In this appendix, we present an exhaustive technical comparison between two identifying restrictions: intertemporal restrictions (IR) and sign restrictions (SR), by thoroughly examining their respective identified sets (IS_{IR}^{ρ} and IS_{SR}^{ρ}). It is important to emphasize that the IR method remains non-redundant when ρ and σ_{21} take opposite signs, which is evident in Figure 1 of the main text, where the main diagonal across all quadrants showcases this distinction. Consequently, the analysis in this appendix focuses on scenarios where ρ and σ_{21} share the same sign.

Our main findings appear summarized in Tables OA1 and OA2 at the bottom of the appendix, which serves as a valuable complement to the results discussed in Section 3.2. The rows labeled as C.11, C.12, C.21, and C.22 in Table OA1 correspond to conditions that determine the sign of the difference of causality spectrum shares $w_{12}^{0} - w_{12}^{\pi}$. Similarly, in Table OA2, the rows labeled as C.31, C.32, C.41, and C.42 represent conditions defining the sign of the difference of causality spectrum shares $w_{22}^{0} - w_{22}^{\pi}$.

To simplify interpretation for Tables OA1 and OA2, we elaborate a color map corresponding to different combinations of specifications that identify potential redundancies. In the tables, light grey cells indicate cases where IR cannot be redundant (but SR can be if $IS_{\rho}^{IR} \subset IS_{\rho}^{SR}$), while dark grey cells represent cases where IS_{IR}^{ρ} may potentially be redundant.

In these tables, all combinations of light grey cells from the same column, result in cases where the IR cannot be redundant. For example, the light grey combinations C.11 in Table OA1 and C.31 in Table OA2 when $\sigma_{21} > 0$, which imply $0 < \rho_{12,2}, \rho_{22,2} < \arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$. In this case, $\rho_{12,2}$ and $\rho_{22,2}$ correspond to R_3 and R_4 depicted in the upper right quarter of all graphs in Figure 1. The position of $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$ is marked with the red dashed arrow with the number 3. If the identified set of IR as in the upper-left or down-right graphs, the combination of IR and SR shrinks the identified set of each restriction when applied separately. If the IR are as in the down-left graph, SR are redundant because $IS_{\rho}^{IR} \subset IS_{\rho}^{SR}$. If the identified set of IR appears as shown in the upper-right graph, IR and SR are not compatible because there are no common solutions that satisfy both sets of restrictions simultaneously.

Moreover, all combinations of light grey and dark grey, or dark grey and light grey from the same column, result in cases where the IR cannot be redundant. For example, the combinations C.11 marked with light grey in Table OA1 ($\sigma_{21} \geq 0$) and C.41 marked with dark grey in Table OA2 ($\sigma_{21} \geq 0$ and $\gamma - F_{11} \geq 0$), imply $0 < \rho_{12,2} < \arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) < \rho_{22,2}$. In Figure 1, the position of $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$ is indicated by a red dashed arrow labeled with the number 2 in the upper right quarter of all graphs in Figure 1 ($\rho_{12,2} = R_3$ and $\rho_{22,2} = R_4$). If IS_{ρ}^{IR} is as in the upper-left and down-left graphs, the joint identified set IS_{ρ}^{SR+IR} is strictly smaller than IS_{ρ}^{SR} or IS_{ρ}^{IR}. If IS_{ρ}^{IR} is as in the upper-right and down-right graphs, IR and SR are not compatible because the intersection of their identified sets is the empty set.

Finally, all combinations of dark grey specifications from both tables produce cases where IR may be redundant if $IS_{\rho}^{SR} \subset IS_{\rho}^{IR}$. For example, the combinations C.21 when for $\sigma_{21} > 0$ and $\alpha - F_{22} \geq 0$ and C.41 when $\sigma_{21} > 0$ and $\gamma - F_{11} \geq 0$, result in $0 < \arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) < \rho_{12,1}, \rho_{22,2}$. In this case, the position of $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$ is indicated by a red dashed arrow labeled with the number 1 in the upper-right quarter of all graphs in Figure 1. If IS_{ρ}^{IR} is represented as shown in the upper-left graph, it leads to redundant IR. Conversely, in all other cases (upper-right, lower-left, and lower-right), IR and SR are not compatible because $IS_{\rho}^{SR} \cap IS_{\rho}^{IR} = \emptyset$.

In sum, the combination of SR and IR restrictions produces an identification set IS_{ρ}^{SR+IR} strictly smaller than IS_{ρ}^{SR} or IS_{ρ}^{IR}, unless

1. ρ and σ_{21} take opposite signs and $\rho(\rho<0,\sigma_{21}\geq 0)\in[R_1,R_2]$ or $\rho(\rho\geq 0,\sigma_{21}<0)\in[R_3,R_4]$, where SR are redundant.

2. ρ and σ_{21} both exhibit the same sign and $\rho(\rho<0,\sigma_{21}<0)\in[R_1,R_2]$ or $\rho(\rho\geq 0,\sigma_{21}\geq 0)\in[R_3,R_4]$ and $\begin{cases}\dfrac{F_{22}\Sigma_{11}}{F_{12}\Sigma_{21}}\geq 1 \quad or \quad \dfrac{F_{22}\Sigma_{11}}{F_{12}\Sigma_{21}}<1,\dfrac{\alpha-F_{22}}{F_{12}\Sigma_{21}}\geq 0\\ \dfrac{F_{11}\Sigma_{22}}{F_{21}\Sigma_{21}}\geq 1 \quad or \quad \dfrac{F_{11}\Sigma_{22}}{F_{21}\Sigma_{21}}<1,\dfrac{\gamma-F_{11}}{F_{21}\Sigma_{21}}\geq 0\end{cases}$, where SR are redundant.

3. ρ and σ_{21} have matching signs and $\rho(\rho<0,\sigma_{21}<0)$ or $\rho(\rho\geq 0,\sigma_{21}\geq 0)$ and $\begin{cases}\dfrac{F_{22}\Sigma_{11}}{F_{12}\Sigma_{21}}<1,\dfrac{\alpha-F_{22}}{F_{12}\Sigma_{21}}<0\\ \dfrac{F_{11}\Sigma_{22}}{F_{21}\Sigma_{21}}<1,\dfrac{\gamma-F_{11}}{F_{21}\Sigma_{21}}<0\end{cases}$, where IR restrictions are redundant.

Demonstrations.

OA1. Proof that $K_{110}^2-K_{11\pi}^2\geq 0$ if and only if $F_{22}\Sigma_{11}-F_{12}\Sigma_{12}\geq 0$[1]

Let us define $R_1=\rho_{12,1}$, $R_2=\rho_{22,1}$, $R_3=\rho_{12,2}$, $R_4=\rho_{22,2}$, and

$$\Sigma=\begin{pmatrix}\Sigma_{11} & \Sigma_{12}\\ \Sigma_{21} & \Sigma_{22}\end{pmatrix}$$

The alignment of the signs for $(K_{110}^2-K_{11\pi}^2)$ and $(K_{110}-K_{11\pi})$ is a result of the positive value of $(K_{11\pi}+K_{110})$ and the equality $(K_{110}^2-K_{11\pi}^2)=(K_{110}-K_{11\pi})(K_{110}+K_{11\pi})$. Now, simplify the expressions for K_{110}^2 and $K_{11\pi}^2$:

[1] Similarly, we can demonstrate that $K_{220}^2-K_{22\pi}^2\geq 0$ if and only if $F_{11}\Sigma_{22}-F_{21}\Sigma_{12}\geq 0$

$$K_{110}^2=\frac{1}{\left[(1-F_{11})(1-F_{22})-F_{12}F_{21}\right]^2}\frac{1}{S_{11}(0)}=$$

$$=\frac{1}{\left[(1-F_{11})(1-F_{22})-F_{12}F_{21}\right]^2}\frac{1}{\frac{1}{\left[(1-F_{11})(1-F_{22})-F_{12}F_{21}\right]^2}\left\{\left[(1-F_{22})\sigma_{11}+F_{12}\sigma_{21}\right]^2+\left[F_{12}\sigma_{22}\right]^2\right\}}=$$

$$=\frac{1}{\left[(1-F_{22})\sigma_{11}+F_{12}\sigma_{21}\right]^2+\left[F_{12}\sigma_{22}\right]^2}$$

and

$$K_{11\pi}^2=\frac{1}{\left[(1+F_{11})(1+F_{22})-F_{12}F_{21}\right]^2}\frac{1}{S_{11}(\pi)}=$$

$$=\frac{1}{\left[(1+F_{11})(1+F_{22})-F_{12}F_{21}\right]^2}\frac{1}{\frac{1}{\left[(1+F_{11})(1+F_{22})-F_{12}F_{21}\right]^2}\left\{\left[(1+F_{22})\sigma_{11}-F_{12}\sigma_{21}\right]^2+\left[-F_{12}\sigma_{22}\right]^2\right\}}=$$

$$=\frac{1}{\left[(1+F_{22})\sigma_{11}-F_{12}\sigma_{21}\right]^2+\left[-F_{12}\sigma_{22}\right]^2}$$

Using these simplifications, $\left(K_{110}^2-K_{11\pi}^2\right)$ becomes

$$K_{110}^2-K_{11\pi}^2=\frac{1}{\left[(1-F_{22})\sigma_{11}+F_{12}\sigma_{21}\right]^2+\left[F_{12}\sigma_{22}\right]^2}-\frac{1}{\left[(1+F_{22})\sigma_{11}-F_{12}\sigma_{21}\right]^2+\left[-F_{12}\sigma_{22}\right]^2}=$$

$$=\frac{\left[(1+F_{22})\sigma_{11}-F_{12}\sigma_{21}\right]^2+\left[-F_{12}\sigma_{22}\right]^2-\left[(1-F_{22})\sigma_{11}+F_{12}\sigma_{21}\right]^2-\left[F_{12}\sigma_{22}\right]^2}{\left\{\left[(1-F_{22})\sigma_{11}+F_{12}\sigma_{21}\right]^2+\left[F_{12}\sigma_{22}\right]^2\right\}\left\{\left[(1+F_{22})\sigma_{11}-F_{12}\sigma_{21}\right]^2+\left[-F_{12}\sigma_{22}\right]^2\right\}}=$$

$$=\frac{4F_{22}\sigma_{11}^2-4\sigma_{11}F_{12}\sigma_{21}}{\left\{\left[(1-F_{22})\sigma_{11}+F_{12}\sigma_{21}\right]^2+\left[F_{12}\sigma_{22}\right]^2\right\}\left\{\left[(1+F_{22})\sigma_{11}-F_{12}\sigma_{21}\right]^2+\left[-F_{12}\sigma_{22}\right]^2\right\}}=$$

$$=\frac{4\left(F_{22}\Sigma_{11}-F_{12}\Sigma_{12}\right)}{\left\{\left[(1-F_{22})\sigma_{11}+F_{12}\sigma_{21}\right]^2+\left[F_{12}\sigma_{22}\right]^2\right\}\left\{\left[(1+F_{22})\sigma_{11}-F_{12}\sigma_{21}\right]^2+\left[-F_{12}\sigma_{22}\right]^2\right\}}\geq 0$$

The denominator of the aforementioned expression is positive, while the numerator is positive if and only if $F_{22}\Sigma_{11}-F_{12}\Sigma_{12}\geq 0$.

OA2. Proof of the results presented in Tables OA1 and OA2

The signs of ρs depend on the signs of the denominators in (19) and (20) in the main text, taking into account that, in all cases, the numerators are positive due to the positive value of σ_{22}.

Furthermore, it is important to recall that $\rho_{12,1}$ and $\rho_{12,2}$, as well as $\rho_{22,1}$ and $\rho_{22,2}$, should exhibit opposite signs. This implies that the denominators in their respective expressions should have opposite signs. Hence, to determine their signs, it is sufficient to identify which denominator is greater. The ρ with the larger denominator will be positive, while the ρ with the smaller denominator will be negative.

Essentially, we assess the redundancy of IR or SR assumptions by comparing the ρs of IR with $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$ of SR. In the case of specifications where $\rho < 0$ and $\sigma_{21} < 0$, we compare $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) < 0$ with negative ρs. Similarly, for $\rho \geq 0$ and $\sigma_{21} \geq 0$, we compare $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) \geq 0$ with positive ρs.

It is worth emphasizing that the signs of $\rho_{12,1}$, $\rho_{12,2}$, $\rho_{22,1}$, and $\rho_{12,2}$ are independent of ρ, but rather depend on the parameters of the reduced-form model. The relationship between these signs can be understood as follows: Conditions c.11, c.12, c.21 and c.22 in Table OA1 determine the signs of $\rho_{12,1}$ and $\rho_{12,2}$ without impacting the signs of $\rho_{22,1}$ and $\rho_{22,2}$. Likewise, conditions c.31, c.32, c.41 and c.42 in Table OA2 establish the signs of $\rho_{22,1}$ and $\rho_{22,2}$ without affecting the signs of $\rho_{12,1}$ and $\rho_{12,2}$.

To clarify the findings, if the condition for the redundancy of the sign restriction is satisfied, we mark the specification in the proof with "**Satisfied**" and the corresponding cell in Table OA1 with light grey color. If the condition for the redundancy of the intertemporal restriction is satisfied, we mark the specification in the proof with "**Not satisfied**" and the corresponding cell in Table OA2 with dark grey color. Thus, light grey cells in the table indicate cases where intertemporal restrictions cannot be redundant (but SR can be), while dark grey cells represent cases where IS_{IR}^{ρ} may potentially be redundant, which occurs only when ρ and σ_{21} are of the same signs.

Condition C1.1: $F_{22}\Sigma_{11} - F_{12}\Sigma_{21} \geq 0$ **and** $F_{12} \geq 0$

Under this condition, both $\alpha, \beta > 0$, $\frac{K_{110} - K_{11\pi}}{K_{110} + K_{11\pi}} = \alpha < \beta = \frac{K_{110} + K_{11\pi}}{K_{110} - K_{11\pi}}$ and $\beta > 1$. Thus, $\frac{\beta - F_{22}}{F_{12}} > 0$ given that $F_{12} \geq 0$, $|F_{22}| < 1$ and $\frac{\alpha - F_{22}}{F_{12}} < \frac{\beta - F_{22}}{F_{12}}$. Regardless of the sign of σ_{21}, the fact that $\sigma_{11} \geq 0$ implies $\sigma_{21} + \sigma_{11}\frac{\alpha - F_{22}}{F_{12}} < \sigma_{21} + \sigma_{11}\frac{\beta - F_{22}}{F_{12}}$. Therefore, $\rho_{12,1} < 0$ and $\rho_{12,2} \geq 0$.

When $\sigma_{21} \geq 0$ the sign restrictions imply that $\frac{\beta - F_{22}}{F_{12}} > 0$, which leads to $\frac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\frac{\beta - F_{22}}{F_{12}}} < \frac{\sigma_{22}}{\sigma_{21}}$ and $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) \geq \rho_{12,2}$. **Satisfied.**

In the case where $\sigma_{21} < 0$, there are two possible scenarios for $\frac{\alpha - F_{22}}{F_{12}}$: it can be either positive or negative However, $\sigma_{21} + \sigma_{11}\frac{\alpha - F_{22}}{F_{12}}$ is negative, given that $\rho_{12,1} < 0$. In the scenario where $\alpha - F_{22} < 0$, then $\frac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\frac{\alpha - F_{22}}{F_{12}}} > \frac{\sigma_{22}}{\sigma_{21}}$ and $\sigma_{21} + \sigma_{11}\frac{\alpha - F_{22}}{F_{12}} < \sigma_{21}$ with both elements being negative implying that $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) < \rho_{12,1}$,. **Satisfied**. However, in the scenario where $\alpha - F_{22} \geq 0$, then $\frac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\frac{\alpha - F_{22}}{F_{12}}} < \frac{\sigma_{22}}{\sigma_{21}}$ since $\sigma_{21} + \sigma_{11}\frac{\alpha - F_{22}}{F_{12}} > \sigma_{21}$, with both elements being negative implying $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) > \rho_{12,1}$. **Not satisfied**.

Condition C1.2: $F_{22}\Sigma_{11} - F_{12}\Sigma_{21} < 0$ **and** $F_{12} < 0$

In this case, both $\alpha, \beta < 0$, with $\alpha > \beta$ and $\beta < -1$. Thus, $\beta - F_{22} < 0$ and $\dfrac{\beta - F_{22}}{F_{12}} > 0$ given that $F_{12} < 0$. As in the previous case, regardless of the sign of σ_{21}, $\dfrac{\alpha - F_{22}}{F_{12}} < \dfrac{\beta - F_{22}}{F_{12}}$ and $\sigma_{21} + \sigma_{11}\dfrac{\alpha - F_{22}}{F_{12}} < \sigma_{21} + \sigma_{11}\dfrac{\beta - F_{22}}{F_{12}}$, which implies that $\rho_{12,2} \geq 0$ and $\rho_{12,1} < 0$.

When $\sigma_{21} \geq 0$, $\rho_{12,2} \geq 0$ with $\dfrac{\beta - F_{22}}{F_{12}} > 0$ gives $\dfrac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\dfrac{\beta - F_{22}}{F_{12}}} < \dfrac{\sigma_{22}}{\sigma_{21}}$, which results in $\arctan\left(\dfrac{\sigma_{22}}{\sigma_{21}}\right) \geq \rho_{12,2}$. **Satisfied**

When $\sigma_{21} < 0$ it happens that $\rho_{12,1} < 0$, but the potential redundancy of the restrictions will depend on the sign of $\alpha - F_{22}$. If $\alpha - F_{22} \geq 0$, $\dfrac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\dfrac{\alpha - F_{22}}{F_{12}}} > \dfrac{\sigma_{22}}{\sigma_{21}}$ and $\arctan\left(\dfrac{\sigma_{22}}{\sigma_{21}}\right) < \rho_{12,1}$. **Satisfied**. If $\alpha - F_{22} < 0$, it is easy to check that **not satisfied**.

Condition C2.1: $F_{22}\Sigma_{11} - F_{12}\Sigma_{21} \geq 0$ **and** $F_{12} < 0$

In this case, $\alpha, \beta > 0$, $\dfrac{K_{110} - K_{11\pi}}{K_{110} + K_{11\pi}} = \alpha < \beta = \dfrac{K_{110} + K_{11\pi}}{K_{110} - K_{11\pi}}$ and $\beta > 1$. Thus, $\beta - F_{22} > 0$, but $\dfrac{\beta - F_{22}}{F_{12}} < 0$ since $F_{12} < 0$. As a consequence, $\dfrac{\alpha - F_{22}}{F_{12}} > \dfrac{\beta - F_{22}}{F_{12}}$ and $\rho_{12,2} < 0$ and $\rho_{12,1} \geq 0$, regardless of the sign of σ_{21}.

When $\sigma_{21} \geq 0$ ($\rho > 0$), if $\alpha - F_{22} \geq 0$, then $\frac{\alpha - F_{22}}{F_{12}} < 0$ and $\frac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\frac{\alpha - F_{22}}{F_{12}}} \geq \frac{\sigma_{22}}{\sigma_{21}}$. As a result, $\arctan\left(\frac{\sigma_{22}}{\sigma_{22}}\right) < \rho_{12,1}$. **Not satisfied**. If $\alpha - F_{22} < 0$, then $\frac{\alpha - F_{22}}{F_{12}} \geq 0$ and $\frac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\frac{\alpha - F_{22}}{F_{12}}} < \frac{\sigma_{22}}{\sigma_{21}}$. As a result, $\arctan\left(\frac{\sigma_{22}}{\sigma_{22}}\right) \geq \rho_{12,1}$. **Satisfied**.

When $\sigma_{21} < 0$ ($\rho < 0$), $\rho_{12,2} < 0$ with $\frac{\beta - F_{22}}{F_{12}} < 0$ and $\sigma_{21} < 0$. Thus, $\frac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\frac{\delta - F_{22}}{F_{12}}} > \frac{\sigma_{22}}{\sigma_{21}}$ with both elements negative. As a result, $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) < \rho_{12,2}$. **Satisfied**.

Condition C2.2: $F_{22}\Sigma_{11} - F_{12}\Sigma_{21} < 0$ **and** $F_{12} \geq 0$

In this case, both $\alpha, \beta < 0$, $\alpha > \beta$ and $\beta < -1$. Thus, $\beta - F_{22} < 0$ and $\beta - F_{22} < \alpha - F_{22}$. As in the previous case, given that $F_{12} \geq 0$, $\frac{\beta - F_{22}}{F_{12}} < \frac{\alpha - F_{22}}{F_{12}}$, $\rho_{12,2} < 0$ and $\rho_{12,1} \geq 0$, regardless of the sign of σ_{21}.

When $\sigma_{21} \geq 0$ ($\rho > 0$) $\rho_{12,1} \geq 0$. However, $\frac{\alpha - F_{22}}{F_{12}}$ can be either positive or negative, depending on the sing of $\alpha - F_{22}$, and always satisfying $\sigma_{21} + \sigma_{11}\frac{\alpha - F_{22}}{F_{12}} \geq 0$. If $\alpha - F_{22} \geq 0$, then $\frac{\alpha - F_{22}}{F_{12}} \geq 0$ and $\frac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\frac{\alpha - F_{22}}{F_{12}}} < \frac{\sigma_{22}}{\sigma_{21}}$. As a result, $\arctan\left(\frac{\sigma_{22}}{\sigma_{22}}\right) \geq \rho_{12,1}$. **Satisfied**. If $\alpha - F_{22} < 0$, then $\frac{\alpha - F_{22}}{F_{12}} < 0$ and $\frac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\frac{\alpha - F_{22}}{F_{12}}} > \frac{\sigma_{22}}{\sigma_{21}}$. As a result, $\arctan\left(\frac{\sigma_{22}}{\sigma_{22}}\right) < \rho_{12,1}$. **Not satisfied**.

When $\sigma_{21} < 0$ ($\rho < 0$), $\rho_{12,2} < 0$ and $\dfrac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\dfrac{\delta - F_{22}}{F_{12}}} > \dfrac{\sigma_{22}}{\sigma_{21}}$. As a result, $\arctan\left(\dfrac{\sigma_{22}}{\sigma_{21}}\right) < \rho_{12,2}$

. **Satisfied**.

Condition C3.1: $F_{11}\Sigma_{22} - F_{21}\Sigma_{21} \geq 0$ **and** $F_{21} \geq 0$

In this case, $\gamma, \delta \geq 0$ and $\gamma < \delta$ with $\delta > 1$. It follows that $\delta - F_{11} \geq 0$ and $\delta - F_{11} \geq \gamma - F_{11}$. If $\sigma_{21} \geq 0$, the only possibility to get a negative ρ is to have $\gamma - F_{11} < 0$. Thus, if $\sigma_{21} \geq 0$, then $\rho_{22,2} \geq 0$ and $\rho_{22,1} < 0$. If $\sigma_{21} < 0$, then $\gamma - F_{11}$ can be either positive or negative. Thus, if $\sigma_{21} < 0$ and $\gamma - F_{11} \geq 0$, then $\dfrac{F_{21}}{\delta - F_{11}} < \dfrac{F_{21}}{\gamma - F_{11}}$ with both elements positive. As a consequence, $\rho_{22,2} < 0$ and $\rho_{22,1} \geq 0$. If $\sigma_{21} < 0$ and $\gamma - F_{11} < 0$, then $\dfrac{F_{21}}{\delta - F_{11}} \geq \dfrac{F_{21}}{\gamma - F_{11}}$ with $\dfrac{F_{21}}{\gamma - F_{11}} < 0$. As a result, $\rho_{22,2} \geq 0$ and $\rho_{22,1} < 0$.

Regarding SR, given $\sigma_{21} \geq 0$ ($\rho \geq 0$) it follows that $\dfrac{F_{21}}{\delta - F_{11}} \geq 0$ and $\dfrac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\dfrac{F_{21}}{\delta - F_{11}}} < \dfrac{\sigma_{22}}{\sigma_{21}}$.

Therefore, we have $\arctan\left(\dfrac{\sigma_{22}}{\sigma_{21}}\right) \geq \rho_{22,2}$. **Satisfied**.

When $\sigma_{21} < 0$ ($\rho < 0$) the results will depend on the sign of $\gamma - F_{11}$. If $\gamma - F_{11} \geq 0$, then $\rho_{22,2} < 0$ with $\dfrac{F_{21}}{\delta - F_{11}} \geq 0$ and $\sigma_{21} + \sigma_{11}\dfrac{F_{21}}{\delta - F_{11}} > \sigma_{21}$ with both elements being negative. Thus, $\dfrac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\dfrac{F_{21}}{\delta - F_{11}}} < \dfrac{\sigma_{22}}{\sigma_{21}}$ and $\arctan\left(\dfrac{\sigma_{22}}{\sigma_{21}}\right) > \rho_{22,2}$. **Not satisfied**. However, if $\gamma - F_{11} < 0$, then

$\rho_{22,1} < 0$ with $\frac{F_{21}}{\gamma - F_{11}} < 0$ and $\sigma_{21} + \sigma_{11}\frac{F_{21}}{\gamma - F_{11}} < \sigma_{21}$ with both elements being negative. In this scenario, $\frac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\frac{F_{21}}{\gamma - F_{11}}} > \frac{\sigma_{22}}{\sigma_{21}}$ and, which implies $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) < \rho_{22,1}$. **Satisfied**.

Condition C3.2: $F_{11}\Sigma_{22} - F_{21}\Sigma_{21} < 0$ **and** $F_{21} < 0$

In this case, $\gamma, \delta < 0$ and $\gamma > \delta$ with $\delta < -1$. It follows that $\delta - F_{11} < 0$ and $\delta - F_{11} < \gamma - F_{11}$. If $\sigma_{21} \geq 0$, then $\rho_{22,2} \geq 0$ since $\frac{F_{21}}{\delta - F_{11}} \geq 0$ ($F_{21} < 0$ and $\delta - F_{11} < 0$). As a result, $\rho_{22,1} < 0$ with $\gamma - F_{11} \geq 0$. If $\sigma_{21} < 0$, then $\gamma - F_{11}$ can be either positive or negative. Thus, if $\sigma_{21} < 0$ and $\gamma - F_{11} \geq 0$, then $\frac{F_{21}}{\gamma - F_{11}} < 0$ and $\frac{F_{21}}{\delta - F_{11}} > \frac{F_{21}}{\gamma - F_{11}}$. As a consequence, $\rho_{22,2} \geq 0$ and $\rho_{22,1} < 0$. If $\sigma_{21} < 0$ and $\gamma - F_{11} < 0$, $\gamma - F_{11} > \delta - F_{11}$ with both element negative, then $\frac{F_{21}}{\delta - F_{11}} < \frac{F_{21}}{\gamma - F_{11}}$, resulting in $\rho_{22,2} < 0$ and $\rho_{22,1} \geq 0$.

When $\sigma_{21} \geq 0$ ($\rho \geq 0$), it happens that $\rho_{22,2} \geq 0$ with $\frac{F_{21}}{\delta - F_{11}} \geq 0$ and $\frac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\frac{F_{21}}{\delta - F_{11}}} < \frac{\sigma_{22}}{\sigma_{21}}$ which implies $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) \geq \rho_{22,2}$. **Satisfied**.

However, when $\sigma_{21} < 0$ ($\rho < 0$), the results will depend on the sign of $\gamma - F_{11}$. If $\gamma - F_{11} \geq 0$, then $\rho_{22,1} < 0$ with $\frac{F_{21}}{\gamma - F_{11}} < 0$ and $\sigma_{21} + \sigma_{11}\frac{F_{21}}{\gamma - F_{11}} < \sigma_{21}$, with both elements being negative. This implies that $\frac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\frac{F_{21}}{\gamma - F_{11}}} > \frac{\sigma_{22}}{\sigma_{21}}$ and $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) < \rho_{22,2}$. **Satisfied**. Conversely, if $\gamma - F_{11} < 0$, then $\rho_{22,2} < 0$ with $\frac{F_{21}}{\delta - F_{11}} \geq 0$ and $\sigma_{21} + \sigma_{11}\frac{F_{21}}{\delta - F_{11}} \geq \sigma_{21}$, with both elements being negative. In this scenario, $\frac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\frac{F_{21}}{\delta - F_{11}}} < \frac{\sigma_{22}}{\sigma_{21}}$ and $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) > \rho_{22,2}$. **Not satisfied**.

Condition C.41: $F_{11}\Sigma_{22} - F_{21}\Sigma_{21} \geq 0$ **and** $F_{21} < 0$

In this case, it follows that $\gamma, \delta \geq 0$ and $\gamma < \delta$ with $\delta > 1$. As a result, $\delta - F_{11} \geq 0$ and $\frac{F_{21}}{\delta - F_{11}} < 0$ since $F_{21} < 0$. When $\sigma_{21} < 0$, the only possibility to have a positive ρ is when $\gamma - F_{11} < 0$. In this scenario, $\frac{F_{21}}{\gamma - F_{11}} \geq 0$ and $\rho_{22,1} \geq 0$ and $\rho_{22,2} < 0$. Conversely, when $\sigma_{21} \geq 0$, the results will depend on the sign of $\gamma - F_{11}$. If $\gamma - F_{11} \geq 0$, then $\frac{F_{21}}{\delta - F_{11}} \geq \frac{F_{21}}{\gamma - F_{11}}$ with both elements being negative, resulting in $\rho_{22,1} < 0$ and $\rho_{22,2} \geq 0$. If $\gamma - F_{11} < 0$, then $\frac{F_{21}}{\delta - F_{11}} \leq \frac{F_{21}}{\gamma - F_{11}}$ with $\frac{F_{21}}{\gamma - F_{11}} \geq 0$, and, as a result, $\rho_{22,1} \geq 0$ and $\rho_{22,2} < 0$.

Regarding the SR, when $\sigma_{21} \geq 0$ ($\rho \geq 0$) it happens that $\rho_{22,2} \geq 0$, with $\frac{F_{21}}{\delta - F_{11}} < 0$. As a result, $\frac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\frac{F_{21}}{\delta - F_{11}}} > \frac{\sigma_{22}}{\sigma_{21}}$ and $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) < \rho_{22,2}$. **Not satisfied** if $\gamma - F_{11} \geq 0$. If $\gamma - F_{11} < 0$, then $\rho_{22,1} \geq 0$ with $\frac{F_{21}}{\gamma - F_{11}} \geq 0$. As a result, $\frac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\frac{F_{21}}{\delta - F_{11}}} < \frac{\sigma_{22}}{\sigma_{21}}$ and $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) > \rho_{22,2}$. **Satisfied** if $\gamma - F_{11} < 0$.

$\sigma_{21} < 0$ ($\rho < 0$): For these specifications, $\rho_{22,1} < 0$ with $\frac{F_{21}}{\delta - F_{11}} < 0$. Given that $\sigma_{21} < 0$, $\frac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\frac{F_{21}}{\delta - F_{11}}} > \frac{\sigma_{22}}{\sigma_{21}}$ with both elements negative, and $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) < \rho_{22,2}$. S**atisfied.**

Condition C.42: $F_{11}\Sigma_{22} - F_{21}\Sigma_{21} < 0$ **and** $F_{21} \geq 0$

In this case, $\gamma, \delta < 0$ and $\gamma > \delta$ with $\delta < -1$. As a result, $\delta - F_{11} < 0$, and $\delta - F_{11} < \gamma - F_{11}$, but given that $F_{21} \geq 0$, it happens that $\frac{F_{21}}{\delta - F_{11}} < 0$. When $\sigma_{21} < 0$, then $\rho_{22,2} < 0$ and $\rho_{22,1} \geq 0$. If $\sigma_{21} \geq 0$, the results will depend on the sign of $\gamma - F_{11}$. If $\gamma - F_{11} \geq 0$, then $\frac{F_{21}}{\delta - F_{11}} < \frac{F_{21}}{\gamma - F_{11}}$ with $\frac{F_{21}}{\delta - F_{11}} < 0$ and $\frac{F_{21}}{\gamma - F_{11}} \geq 0$. As a result, $\rho_{22,1} \geq 0$ and $\rho_{22,2} < 0$. If $\gamma - F_{11} < 0$, then $\frac{F_{21}}{\delta - F_{11}} \geq \frac{F_{21}}{\gamma - F_{11}}$ with both elements being negative. This implies $\rho_{22,1} < 0$ and $\rho_{22,2} \geq 0$.

Regarding SR, when $\sigma_{21} \geq 0$ ($\rho \geq 0$) and $\gamma - F_{11} \geq 0$, then $\rho_{22,1} \geq 0$ with $\frac{F_{21}}{\gamma - F_{11}} \geq 0$. As a result, $\frac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\frac{F_{21}}{\gamma - F_{11}}} < \frac{\sigma_{22}}{\sigma_{21}}$ and $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) > \rho_{22,1}$. **Satisfied**. However, when $\gamma - F_{11} < 0$, then $\rho_{22,2} \geq 0$ with $\frac{F_{21}}{\delta - F_{11}} < 0$. As a result, $\frac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\frac{F_{21}}{\delta - F_{11}}} > \frac{\sigma_{22}}{\sigma_{21}}$ and $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) < \rho_{22,2}$. **Not satisfied**

Conversely, when $\sigma_{21} < 0$ ($\rho < 0$) it happens that $\rho_{22,2} < 0$ and $\frac{F_{21}}{\delta - F_{11}} < 0$. Given that $\sigma_{21} < 0$, it happens that $\frac{\sigma_{22}}{\sigma_{21} + \sigma_{11}\frac{F_{21}}{\delta - F_{11}}} > \frac{\sigma_{22}}{\sigma_{21}}$, with both elements being negative, and $\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) < \rho_{22,2}$. S**atisfied.**

OA3. Redundant sign restrictions

To conclude, we will now outline the scenarios in which SR are redundant, classifying them according to whether ρ and σ_{21} exhibit the same or opposite signs.Firstly, when ρ and σ_{21} have opposite signs, the following list outlines the specifications that result in redundant SR:

- The process is specified by C1.1, C1.2, C3.1 and C3.2: $\rho \geq 0$ and $\sigma_{21} < 0$, $\gamma - F_{11} \geq 0$ in C3, with limits of the interval in $\rho_{12,2}$ and $\rho_{22,1}$ => $\left|\rho_{12,2} - \rho_{22,1}\right| < \frac{\pi}{2}$; $\rho \geq 0$ and $\sigma_{21} < 0$, $\gamma - F_{11} < 0$ in C3, with limits of the interval in $\rho_{12,2}$ and $\rho_{22,2}$ => $\left|\rho_{12,2} - \rho_{22,2}\right| < \frac{\pi}{2}$; $\rho < 0$ and $\sigma_{21} \geq 0$ with limits of the interval in $\rho_{12,1}$ and $\rho_{22,1}$ => $\left|\rho_{12,1} - \rho_{22,1}\right| < \frac{\pi}{2}$.
- The process is specified by C1.1, C1.2, C4.1 and C4.2: $\rho \geq 0$ and $\sigma_{21} < 0$ with limits of the interval in $\rho_{12,2}$ and $\rho_{22,1}$ => $\left|\rho_{12,2} - \rho_{22,1}\right| < \frac{\pi}{2}$; $\rho < 0$ and $\sigma_{21} \geq 0$, $\frac{\gamma - F_{11}}{F_{21}} \geq 0$ in C4, with limits of the interval in $\rho_{12,1}$ and $\rho_{22,2}$ => $\left|\rho_{12,1} - \rho_{22,2}\right| < \frac{\pi}{2}$; $\rho < 0$ and $\sigma_{21} \geq 0$, $\frac{\gamma - F_{11}}{F_{21}} < 0$ in C4, with limits of the interval in $\rho_{12,1}$ and $\rho_{22,1}$ => $\left|\rho_{12,1} - \rho_{22,1}\right| < \frac{\pi}{2}$
- The process is specified by C2 and C3: $\rho \geq 0$ and $\sigma_{21} < 0$, $\frac{\gamma - F_{11}}{F_{21}} \geq 0$ in C3 with limits of the interval in $\rho_{12,1}$ and $\rho_{22,1}$ => $\left|\rho_{12,1} - \rho_{22,1}\right| < \frac{\pi}{2}$; $\rho \geq 0$ and $\sigma_{21} < 0$, $\frac{\gamma - F_{11}}{F_{21}} < 0$ in C3 with limits of the interval in $\rho_{12,1}$ and $\rho_{22,2}$ => $\left|\rho_{12,1} - \rho_{22,2}\right| < \frac{\pi}{2}$; $\rho < 0$ and $\sigma_{21} \geq 0$, with limits of the interval in $\rho_{12,2}$ and $\rho_{22,1}$ => $\left|\rho_{12,1} - \rho_{22,2}\right| < \frac{\pi}{2}$.

- The process is specified by C2.1, C2.2, C4.1 and C4.2: $\rho > 0$ and $\sigma_{21} < 0$ with limits of the interval in $\rho_{12,1}$ and $\rho_{22,1}$ => $\left|\rho_{12,1} - \rho_{22,1}\right| < \frac{\pi}{2}$; $\rho < 0$ and $\sigma_{21} \geq 0$, $\frac{\gamma - F_{11}}{F_{21}} \geq 0$ in C4 with limits of the interval in $\rho_{12,2}$ and $\rho_{22,1}$ => $\left|\rho_{12,2} - \rho_{22,1}\right| < \frac{\pi}{2}$; $\rho < 0$ and $\sigma_{21} \geq 0$, $\frac{\gamma - F_{11}}{F_{21}} < 0$ in C4 with limits of the interval in $\rho_{12,2}$ and $\rho_{22,2}$ => $\left|\rho_{12,2} - \rho_{22,2}\right| < \frac{\pi}{2}$.

- ρ and σ_{21} are of the same sign and $\begin{cases} \frac{F_{22}\Sigma_{11}}{F_{12}\Sigma_{21}} \geq 1 \\ \frac{F_{11}\Sigma_{22}}{F_{21}\Sigma_{21}} \geq 1 \end{cases}$ or, $\begin{cases} \frac{F_{22}\Sigma_{11}}{F_{12}\Sigma_{21}} < 1, \frac{\alpha - F_{22}}{F_{12}\Sigma_{21}} \geq 0 \\ \frac{F_{11}\Sigma_{22}}{F_{21}\Sigma_{21}} < 1, \frac{\gamma - F_{11}}{F_{21}\Sigma_{21}} \geq 0 \end{cases}$.

Secondly, when ρ and σ_{21} are of the same, SR are redundant if $\begin{cases} \rho\left(\rho < 0, \sigma_{21} < 0\right) \in \left[R_1, R_2\right] \\ R_1, R_2 > \arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) \end{cases}$ or $\begin{cases} \rho\left(\rho \geq 0, \sigma_{21} \geq 0\right) \in \left[R_3, R_4\right] \\ R_3, R_4 < \arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) \end{cases}$. This occurs in one of the two following scenarios:

(i) $\begin{cases} \frac{F_{22}\Sigma_{11} - F_{12}\Sigma_{21}}{F_{12}\Sigma_{21}} \geq 0 \\ \frac{F_{11}\Sigma_{22} - F_{21}\Sigma_{21}}{F_{21}\Sigma_{21}} \geq 0 \end{cases}$

Simplifying the expressions: $\frac{F_{22}\Sigma_{11} - F_{12}\Sigma_{21}}{F_{12}\Sigma_{21}} = \frac{F_{22}\Sigma_{11}}{F_{12}\Sigma_{21}} - 1 \geq 0$ or $\frac{F_{22}\sigma_{11}}{F_{12}\sigma_{21}} \geq 1$ and

$\frac{F_{11}\Sigma_{22} - F_{21}\Sigma_{21}}{F_{21}\Sigma_{21}} = \frac{F_{11}\Sigma_{22}}{F_{21}\Sigma_{21}} - 1 \geq 0$ or $\frac{F_{11}\Sigma_{22}}{F_{21}\Sigma_{21}} \geq 1$.

(ii) The condition (i) is not satisfied, but

$$\begin{cases} \frac{F_{22}\Sigma_{11}}{F_{12}\Sigma_{21}} < 1, \frac{\alpha - F_{22}}{F_{12}\Sigma_{21}} \geq 0 \\ \frac{F_{11}\Sigma_{22}}{F_{21}\Sigma_{21}} < 1, \frac{\gamma - F_{11}}{F_{21}\Sigma_{21}} \geq 0 \end{cases} \text{or} \begin{cases} \frac{F_{22}\Sigma_{11}}{F_{12}\Sigma_{21}} < 1, \frac{\alpha - F_{22}}{F_{12}\Sigma_{21}} < 0 \\ \frac{F_{11}\Sigma_{22}}{F_{21}\Sigma_{21}} < 1, \frac{\gamma - F_{11}}{F_{21}\Sigma_{21}} < 0 \end{cases}.$$

Table OA1. Signs of ρ for intertemporal restrictions on $w_{12}^{0} - w_{12}^{\pi}$

<table>
<tr><th rowspan="2"></th><th colspan="2">$\sigma_{21} \geq 0$ ($\Sigma_{21} \geq 0$)</th><th colspan="2">$\sigma_{21} < 0$ ($\Sigma_{21} < 0$)</th></tr>
<tr><th>$\alpha - F_{22} \geq 0$</th><th>$\alpha - F_{22} < 0$</th><th>$\alpha - F_{22} \geq 0$</th><th>$\alpha - F_{22} < 0$</th></tr>
<tr><td>C.11: $F_{22}\Sigma_{11} - F_{12}\Sigma_{21} \geq 0$
$F_{12} \geq 0$</td><td colspan="2" rowspan="2">$\rho_{12,1} < 0$,
$\rho_{12,2} \geq 0$,
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) \geq \rho_{12,2}$</td><td>$\rho_{12,1} < 0$, $\rho_{12,2} \geq 0$
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$
$\geq \rho_{12,1}$</td><td>$\rho_{12,1} < 0$,
$\rho_{12,2} \geq 0$,
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$
$< \rho_{12,1}$</td></tr>
<tr><td>C.12: $F_{22}\Sigma_{11} - F_{12}\Sigma_{21} < 0$
$F_{12} < 0$</td><td>$\rho_{12,1} < 0$, $\rho_{12,2} \geq 0$,
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$
$< \rho_{12,1}$</td><td>$\rho_{12,1} < 0$,
$\rho_{12,2} \geq 0$,
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$
$\geq \rho_{12,1}$</td></tr>
<tr><td>C.21: $F_{22}\Sigma_{11} - F_{12}\Sigma_{21} \geq 0$
$F_{12} < 0$</td><td>$\rho_{12,1} \geq 0$,
$\rho_{12,2} < 0$,
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$
$< \rho_{12,1}$</td><td>$\rho_{12,1} \geq 0$,
$\rho_{12,2} < 0$,
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$
$\geq \rho_{12,1}$</td><td colspan="2" rowspan="2">$\rho_{12,1} \geq 0$,
$\rho_{12,2} < 0$,
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) < \rho_{12,2}$</td></tr>
<tr><td>C.22: $F_{22}\Sigma_{11} - F_{12}\Sigma_{21} < 0$
$F_{12} \geq 0$</td><td>$\rho_{12,1} \geq 0$,
$\rho_{12,2} < 0$,
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$
$\geq \rho_{12,1}$</td><td>$\rho_{12,1} \geq 0$,
$\rho_{12,2} < 0$,
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$
$< \rho_{12,1}$</td></tr>
</table>

Table OA2. Signs of ρ for intertemporal restrictions on $w_{22}^{0} - w_{22}^{\pi}$.

<table>
<tr><td rowspan="2"></td><td colspan="2">$\sigma_{21} \geq 0$ ($\Sigma_{21} \geq 0$)</td><td colspan="2">$\sigma_{21} < 0$ ($\Sigma_{21} < 0$)</td></tr>
<tr><td>$\gamma - F_{11} \geq 0$</td><td>$\gamma - F_{11} < 0$</td><td>$\gamma - F_{11} \geq 0$</td><td>$\gamma - F_{11} < 0$</td></tr>
<tr><td>C.31: $F_{11}\Sigma_{22} - F_{21}\Sigma_{21} \geq 0$
$F_{21} \geq 0$</td><td colspan="2" rowspan="2">$\rho_{22,1} < 0$,
$\rho_{22,2} \geq 0$,
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) \geq \rho_{22,2}$</td><td>$\rho_{22,1} \geq 0$, $\rho_{22,2} < 0$,
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$
$\geq \rho_{22,2}$</td><td>$\rho_{22,1} < 0$,
$\rho_{22,2} \geq 0$,
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$
$< \rho_{22,1}$</td></tr>
<tr><td>C.32: $F_{11}\Sigma_{22} - F_{21}\Sigma_{21} < 0$
$F_{21} < 0$</td><td>$\rho_{22,1} < 0$, $\rho_{22,2} \geq 0$,
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$
$< \rho_{22,1}$</td><td>$\rho_{22,1} \geq 0$,
$\rho_{22,2} < 0$,
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$
$\geq \rho_{22,2}$</td></tr>
<tr><td>C.41: $F_{11}\Sigma_{22} - F_{21}\Sigma_{21} \geq 0$
$F_{21} < 0$</td><td>$\rho_{22,1} < 0$,
$\rho_{22,2} \geq 0$,
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$
$< \rho_{22,2}$</td><td>$\rho_{22,1} \geq 0$,
$\rho_{22,2} < 0$,
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$
$\geq \rho_{22,1}$</td><td colspan="2" rowspan="2">$\rho_{22,1} \geq 0$,
$\rho_{22,2} < 0$,
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right) < \rho_{22,2}$</td></tr>
<tr><td>C.42: $F_{11}\Sigma_{22} - F_{21}\Sigma_{21} < 0$
$F_{21} \geq 0$</td><td>$\rho_{22,1} \geq 0$,
$\rho_{22,2} < 0$,
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$
$\geq \rho_{22,1}$</td><td>$\rho_{22,1} < 0$,
$\rho_{22,2} \geq 0$,
$\arctan\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$
$< \rho_{22,2}$</td></tr>
</table>