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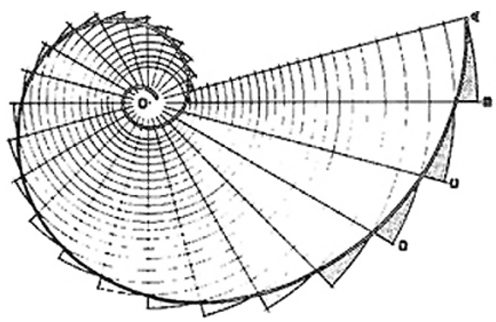

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Switched symplectic graphs and their 2-ranks

Aida Abiad[1]

We apply Godsil-McKay switching to the symplectic graphs over $\mathbb{F}_2$ with at least 63 vertices and prove that the 2-rank of (the adjacency matrix of) the graph increases after switching. This shows that the switched graph is a new strongly regular graph with parameters $(2^{2\nu}-1, 2^{2\nu-1}, 2^{2\nu-2}, 2^{2\nu-2})$ and 2-rank $2\nu+2$ when $\nu \geq 3$. For the symplectic graph on 63 vertices we investigate repeated switching by computer and find many new strongly regular graphs with the above parameters for $\nu = 3$ with various 2-ranks. Using these results and a recursive construction method for the symplectic graph from Hadamard matrices, we obtain several graphs with the above parameters, but different 2-ranks for every $\nu \geq 3$.

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Sobre la clasificación de 3-politopos reticulares

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En esta charla expondré el trabajo que hemos desarrollado en vistas a una enumeración algorítmica de 3-politopos reticulares (módulo equivalencia unimodular).

Consideramos dos parámetros de politopos reticulares: la *anchura* reticular (mínima distancia entre dos hiperplanos paralelos que acotan al politopo) y su *tamaño* (número de puntos enteros en el politopo). Decimos que un politopo reticular es *grueso* si su anchura es mayor que uno. Diremos que un politopo grueso es *minimal* si ninguno de sus subpolitopos propios es grueso, y *quasiminimal* si a lo sumo uno de sus subpolitopos propios maximales (respecto a la inclusión) lo es.

Probamos lo siguiente:

- Para cada n, el número de 3-politopos reticulares gruesos de tamaño n es finito.
- Excepto por una cantidad finita, todos los 3-politopos (quasi)minimales proyectan sobre uno de entre cinco polígonos reticulares de una manera muy específica. Los politopos que están entre las excepciones tienen tamaño a lo sumo 11.

Utilizamos primero estas caracterizaciones para clasificar 3-politopos (quasi)minimales de un tamaño dado, y luego presentamos un algoritmo que elabora la lista completa de 3-politopos reticulares gruesos de tamaño n, a partir de la de tamaño $n-1$. Como punto de partida, clasificamos los 3-politopos de tamaños 5 y 6.

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Polytopes and Plethysm

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Plethysm coefficients are important structural constants in the representation theory of the symmetric groups and general linear groups. Remarkably, some sequences of plethysm coefficients stabilize (they are ultimately constants). These results were proved by Brion in [2], and Thibon and Carré in [1]. We have given a new proof of those stability properties. Our new proofs are purely combinatorial: we decompose plethysm coefficients as a alternating sum of terms counting integer points in polytopes, and exhibit bijections between these sets of integer points. More details about the results can be found in [3].

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On the Cartesian sum of graphs and hyperbolicity

Amauris de la Cruz[1], **Walter Carballosa**[2], **José M. Rodríguez**[1]

If X is a geodesic metric space and $x_1, x_2, x_3 \in X$, a *geodesic triangle* $T = \{x_1, x_2, x_3\}$ is the union of the three geodesics $[x_1x_2]$, $[x_2x_3]$ and $[x_3x_1]$ in X. The space X is d-*hyperbolic* (in the Gromov sense) if any side of T is contained in a d-neighborhood of the union of the two other sides, for every geodesic triangle T in X. If X is hyperbolic, we denote by $d(X)$ the sharp hyperbolicity constant of X, i.e. $d(X) = \inf\{d \geq 0 : \ X \text{ is d-hyperbolic }\}$. Some previous works characterize the hyperbolic product graphs (for the Cartesian, strong, join, corona and lexicographic products) in terms of properties of the factor graphs. In this paper we characterize the hyperbolic product graphs for the Cartesian sum $G_1 \oplus G_2$: $G_1 \oplus G_2$ is always hyperbolic, unless either G_1 or G_2 is the trivial graph (the graph with a single vertex); if G_1 or G_2 is the trivial graph, then $G_1 \oplus G_2$ is hyperbolic if and only if G_2 or G_1 is hyperbolic, respectively. Besides, if $t \notin \{5/4, 3/2\}$ we characterize the Cartesian sums with $d(G_1 \oplus G_2) = t$ in a very simple way; also, we characterize the Cartesian sums with $d(G_1 \oplus G_2) = 5/4$ and with $d(G_1 \oplus G_2) = 3/2$. We obtain the sharp inequalities $1 \leq d(G_1 \oplus G_2) \leq 3/2$ for every non-trivial graphs G_1, G_2. Furthermore, we obtain simple formulae for the hyperbolicity constant of the Cartesian sum of many graphs. Finally, we prove the inequalities $3/2 \leq d(\overline{G_1 \oplus G_2}) \leq 2$ for the complement graph of $G_1 \oplus G_2$ for every G_1, G_2 with $\text{mín}\{diamV(G_1), diamV(G_2)\} \geq 3$.

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Distance mean-regular graphs

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We introduce the concept of distance mean-regular graph, which can be seen as a generalization of both vertex-transitive and distance-regular graphs. A graph $\Gamma = (V, E)$ with diameter D is *distance mean-regular* when, for given $u \in V$, the averages of the intersection numbers $a_i(u, v)$, $b_i(u, v)$, and $c_i(u, v)$ (defined as usual), computed over all vertices v at distance $i = 0, 1, \ldots, D$ from u, do not depend on u. In this work we study some properties and characterizations of these graphs. For instance, it is shown that a distance mean-regular graph is always distance degree-regular, but the converse does not hold, and give a condition for the converse to be true. Some algebraic and spectral properties of distance mean-regular graphs are also investigated. We show that, for distance mean regular-graphs, the role of the distance matrices of distance-regular graphs is played for the so-called *distance mean-regular* matrices. Moreover, these matrices are computed from a sequence of orthogonal polynomials evaluated at A, the adjacency matrix of Γ.

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Cotas para la constante de hiperbolicidad de Gromov

Verónica Hernández[1], **Domingo Pestana**[2], **José M. Rodríguez**[3]

Si X es un espacio métrico geodésico y $x_1, x_2, x_3 \in X$, un triángulo geodésico $T = \{x_1, x_2, x_3\}$ es la unión de las tres geodésicas $[x_1x_2]$, $[x_2x_3]$, y $[x_3x_1]$ en X. El espacio X es δ-hiperbólico en el sentido de Gromov si cualquier lado de T está contenido en una δ-vecindad de la unión de los otros dos lados, para todo triángulo geodésico T en X. Si X is hiperbólico, denotamos por $\delta(X)$ a la constante de hiperbolicidad óptima de X, i.e. $\delta(X) = \inf\{\delta \geq 0 : \quad X \text{ es } \delta\text{-hiperbólico}\}$. Calcular la constante de hiperbolicidad es un problema generalmente muy difícil. En consecuencia, resulta natural intentar acotar la constante de hiperbolicidad en función de algunos parámetros del grafo. Denotamos por $\mathcal{G}(n,m)$ al conjunto de grafos G con n vértices y m aristas, y tales que cada arista tiene longitud 1. En este trabajo estimamos $A(n,m) := \text{mín}\{\delta(G) \mid G \in \mathcal{G}(n,m)\}$ y $B(n,m) := \text{máx}\{\delta(G) \mid G \in \mathcal{G}(n,m)\}$. En particular, obtenemos buenas cotas para $B(n,m)$, y calculamos el valor preciso $A(n,m)$ para todos los valores de n y m. Además, aplicamos estos resultados a grafos aleatorios. De forma adicional, obtenemos una cota para el tamaño de un grafo en función de su diámetro y su orden.

Subdivisiones Recursivamente Regulares

Rafel Jaume[1], **Günter Rote**[1]

Una subdivisión poliédrica regular es aquella que coincide con la proyección de (las caras inferiores de) un politopo. Las subdivisiones regulares exhiben buenas propiedades y han sido extensamente estudiadas. En este trabajo [**?**], presentamos las subdivisiones recursivamente regulares: subdivisiones que pueden ser divididas mediante una subdivisión regular en partes regulares o recursivamente regulares. Mostramos ejemplos y contraejemplos de estas subdivisiones en el plano. Además, estudiamos algunas propiedades que comparten con las subdivisiones regulares, como el reconocimiento eficiente o la ausencia de ciclos de visibilidad, y algunas que no mantienen, como la conexión por *flips*. En el proceso, estudiamos también la subdivisión regular más fina que es refinada por una subdivisión dada. Finalmente, proponemos una aplicación de los resultados a la teoría de tensegridades y a un problema de iluminación (o recubrimiento) propuesto en 1981 [1, 3].

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The Simultaneous (Strong) Metric Dimension of Graph Families

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In a graph $G = (V, E)$, a vertex $v \in V$ is said to *distinguish* two vertices x and y if $d_G(v, x) \neq d_G(v, y)$, where $d_G(x, y)$ is the length of a shortest path between u and v. Likewise, a vertex x is said to *strongly distinguish* two different vertices u and v if there exists a shortest $u - x$ path containing v or there exists a shortest $v - x$ path containing u, *i.e.* $d_G(u, x) = d_G(u, v) + d_G(v, x)$ or $d_G(v, x) = d_G(u, v) + d_G(u, x)$. A set $S \subset V$ is said to be a (*strong*) *metric generator* for G if any pair of different vertices of G is (strongly) distinguished by some element of S. A minimum (strong) metric generator is called a (*strong*) *metric basis*, and its cardinality the (*strong*) *metric dimension* of G, denoted by $\dim(G)$ ($\dim_s(G)$) [1, 2, 3].

Given a family $\mathcal{G} = \{G_1, G_2, ..., G_k\}$ of (not necessarily edge-disjoint) connected graphs $G_i = (V, E_i)$ on a common vertex set V (the union of whose edge sets is not necessarily the complete graph), we define a *simultaneous* (*strong*) *metric generator* for $\mathcal{G}$ as a set $S \subset V$ such that S is simultaneously a (strong) metric generator for each G_i. We say that a minimum simultaneous (strong) metric generator for $\mathcal{G}$ is a *simultaneous* (*strong*) *metric basis* of $\mathcal{G}$, and its cardinality the *simultaneous* (*strong*) *metric dimension* of $\mathcal{G}$, denoted by $Sd(\mathcal{G})$ ($Sd_s(\mathcal{G})$) or explicitly by $Sd(G_1, G_2, ..., G_k)$ ($Sd_s(G_1, G_2, ..., G_k)$) [4, 5].

We investigate the properties of these simultaneous resolvability parameters. In particular, we determine their general bounds and give exact values or tight bounds for specific families. Additionally, we analyse the relations between both notions of simultaneous resolvability. Finally, we show that computing the simultaneous (strong) metric dimension is NP-hard, even for families composed by graphs whose individual (strong) metric dimension is easily computable.

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Subgraph statistics in subcritical graph classes

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In the last years, a lot of attention has been devoted to the study of random graphs from constrained classes. A prominent example of such families are the so-called subcritical graph classes, which covers, among others, trees, outerplanar graphs and series-parallel graphs. In this talk we study the following problem: given a fixed graph H, and a subcritical graph class G, how many copies of H (as subgraphs) are there in a uniformly at random graph of size n in G? We show that in a general context such number follows a normal distribution. These results widely generalizes known different known results concerning the number of pending copies of a given subgraph [1]. As a case study, we get explicit constants for the important family of series-parallel graphs.

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Small values of the hyperbolicity constant in graphs

Omar Rosario Cayetano[1], **José Manuel Rodríguez**[1], **José M. Sigarreta**[2], **Sergio Bermudo Navarrete**[3]

If X is a geodesic metric space and $x_1, x_2, x_3 \in X$, a *geodesic triangle* $T = \{x_1, x_2, x_3\}$ is the union of the three geodesics $[x_1x_2]$, $[x_2x_3]$ and $[x_3x_1]$ in X. The space X is δ-*hyperbolic* (in the Gromov sense) if any side of T is contained in a δ-neighborhood of the union of the two other sides, for every geodesic triangle T in X. We denote by $\delta(X)$ the sharpest hyperbolicity constant of X, i.e., $\delta(X) := \inf\{\delta \geq 0 : X \text{ is } \delta\text{-hyperbolic}\}$. In the study of any parameter on graphs it is natural to study the graphs for which this parameter has small values. In this work we study the graphs (with every edge of length k) with small hyperbolicity constant, i.e., the graphs which are like trees (in the Gromov sense). In this work we obtain simple characterizations of the graphs G with $\delta(G) = k$ and $\delta(G) = \frac{5k}{4}$ (the case $\delta(G) < k$ is known). Also, we give a necessary condition in order to have $\delta(G) = \frac{3k}{2}$ (we know that $\delta(G)$ is a multiple of $\frac{k}{4}$). Although it is not possible to obtain bounds for the diameter of graphs with small hyperbolicity constant, we obtain such bounds for the effective diameter if $\delta(G) < \frac{3k}{2}$. This is the best possible result, since we prove that it is not possible to obtain similar bounds if $(G) \geq \frac{3k}{2}$.

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Representación Ortogonal de Grafos

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En este trabajo se presenta una variante del problema *representación ortogonal de grafos*, centrado en las representaciones ortogonales *fieles*. El interés por estas representaciones surge de los fundamentos de la mecánica cuántica. El *rango ortogonal* de un grafo es la mínima dimensión d para la que el grafo puede ser representado ortogonalmente de manera fiel en un espacio vectorial $\mathbb{R}^d$. Aspectos de este problema fueron estudiados por Lovász y otros autores.

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Grafos 3-regulares, snarks y multipolos

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Los problemas de coloración de aristas tienen interés práctico y además pueden relacionarse con cuestiones teóricas importantes como la cuestión P vs. NP, la factorización de números enteros y, ya dentro de la teoría de grafos, varias conjeturas acerca de flujos y recubrimientos por ciclos. Incluso en el caso de los grafos 3-regulares, o cúbicos, topamos rápidamente con problemas difíciles. En la vertiente práctica, veremos ejemplos de que la coloración de aristas no es necesariamente intratable para grafos relativamente grandes, ya sean aleatorios o muy estructurados. En la vertiente teórica, seguiremos algunos pasos en el recorrido hacia una comprensión un poco mejor de los grafos cúbicos cuyas aristas no se pueden colorear con tres colores, conocidos como "snarks", mediante el estudio de componentes con extremos libres llamadas "multipolos".

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