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RINGS WHOSE PURE-INJECTIVE RIGHT MODULES ARE DIRECT SUMS OF LIFTING MODULES

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Let R be an associative ring with identity. A module M is called *lifting* if for every submodule N of M there exists a direct sum decomposition $M = M_1 \oplus M_2$ with $M_1 \leq N$ and $N \cap M_2$ small in M_2. A module M is called *extending* if every submodule of M is essential in a direct summand of M. Recall that a ring R is said to be *of finite type* when there exists a finite set of indecomposable right R-modules such that any other right module is isomorphic to a direct sum of copies of them. In this case, R is left and right artinian and there also exists a finite set of indecomposable left R-modules such that any other left module is isomorphic to a direct sum of copies of them. And a ring R is *of right local type* when every indecomposable right R-module is local. In this talk we announce the following:

Theorem: The following are equivalent for a ring R:

1. Every right R-module is a direct sum of lifting modules.
2. Every pure-injective right R-module is a direct sum of lifting modules.
3. R is of finite type and right local type.

Corollary: The following are equivalent for a ring R:

1. R is both sided serial and artinian.
2. Every left and every right R-module is a direct sum of lifting modules.
3. Every left and every right pure-injective R-module is a direct sum of lifting modules.
4. Every left and every right R-module is a direct sum of extending modules.

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