## Some progress on the polynomial Dunford-Pettis property

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A Banach space $E$ has the Dunford-Pettis property (DPP, for short) if every weakly compact (linear) operator on $E$ is completely continuous. The $\mathscr{L}_1$ and the $\mathscr{L}_\infty$-spaces have the DPP. In 1979 R. A. Ryan proved that $E$ has the DPP if and only if every weakly compact polynomial on $E$ is completely continuous.
Every $k$-homogeneous (continuous) polynomial $P \in \mathscr{P}({}^kE, F)$ between Banach spaces $E$ and $F$ admits an extension $\widetilde{P} \in \mathscr{P}({}^kE^{**}, F^{**})$ called the Aron-Berner extension. The Aron-Berner extension of every weakly compact polynomial $P \in \mathscr{P}({}^kE, F)$ is $F$-valued, that is, $\widetilde{P}(E^{**}) \subseteq F$, but there are nonweakly compact polynomials with $F$-valued Aron-Berner extension.
We strengthen Ryan's result by showing that $E$ has the DPP if and only if every polynomial $P \in \mathscr{P}({}^kE, F)$ with $F$-valued Aron-Berner extension is completely continuous whenever the closed unit ball $B_{F^*}$ is weak-star sequentially compact. This gives a partial answer to a question raised in 2003 by I. Villanueva and J. M. Gutiérrez. They proved the result for spaces $E$ such that every operator from $E$ into the dual $E^*$ is weakly compact, but the question remained open for other spaces such as the $\mathscr{L}_1$-spaces.
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