An open question about diametral dimensions

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Introduction: Diametral dimension(s)

Some partial results

Non-Fréchet spaces

Kolmogorov's diameters

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Important property

If U is absolutely convex, V is precompact with respect to U iff

$$\delta_n(V,U) \to 0 \text{ if } n \to \infty.$$

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Properties

- Δ is a topological invariant: if $E \cong F$, then $\Delta(E) = \Delta(F)$.
- If E is an l.c.s.,
  - if E is not Schwartz, $\Delta(E) = c_0$;
  - if E is Schwartz, $l_\infty \subseteq \Delta(E)$.

Another diametral dimension

Definition

If E is a t.v.s. and $\mathcal{U}$ is basis of 0-nghbs,

$$\Delta_b(E) := \left\{ \xi \in \mathbb{C}^{\mathbb{N}_0} : \forall U \in \mathcal{U},\ \forall B \text{ bounded},\ \xi_n \delta_n(B, U) \to 0 \right\}.$$

Reminder: $\Delta(E) = \left\{ \xi \in \mathbb{C}^{\mathbb{N}_0} : \forall U \in \mathcal{U}, \exists V \in \mathcal{U} \text{ s.t. } \xi_n \delta_n(V, U) \to 0 \right\}.$

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Remark: $\Delta(E) \subseteq \Delta_b(E)$.

Open question (Mityagin):

Do we have $\Delta(E) = \Delta_b(E)$ in Fréchet spaces?

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A new characterization

Let E a Fréchet space.

Proposition

- If E is not Montel, $\Delta_b(E) = c_0$.
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• If E is not Montel, $\Delta_b(E) = c_0$. • If E is Montel, $l_\infty \subseteq \Delta_b(E)$.	• If E is not Schwartz, $\Delta(E) = c_0$. • If E is Schwartz, $l_\infty \subseteq \Delta(E)$.

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Consequences

- If E is not Montel: $\Delta(E) = \Delta_b(E) = c_0$.
- If E is Montel but not Schwartz: $\Delta(E) = c_0 \subsetneq \Delta_b(E)$.

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- If E is Montel but not Schwartz: $\Delta(E) = c_0 \subsetneq \Delta_b(E)$.

New open question:

Do we have $\Delta(E) = \Delta_b(E)$ in Fréchet-Schwartz spaces?

Main result

Notations

$$\Delta(E) = \left\{\xi \in \mathbb{C}^{\mathbb{N}_0} : \forall U \in \mathcal{U},\ \exists V \in \mathcal{U},\ \text{s.t. } (\xi_n \delta_n(V,U))_n \in c_0\right\},$$

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Theorem (2016, L.D., L. Frerick, J. Wengenroth)

If E is Fréchet-Schwartz, then $\Delta^{\infty}(E) = \Delta_b^{\infty}(E)$.

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We fix $(U_k)_k$ a decreasing basis of absolutely convex 0-nghbs in E

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So $C + 1 \geq |\xi_{n_k}|\delta_{n_k}(U_k, U_m) > k$. Impossible! ■

Main result

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- **Köthe-Schwartz** sequence spaces (2017, F. Bastin, L.D.).

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Construction of counterexamples

Key idea

When every bounded set is finite-dimensional, then $\Delta_b(E) = \mathbb{C}^{\mathbb{N}_0}$.

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Construction of Schwartz spaces E with only finite-dimensional bounded sets and $\Delta(E) \neq \mathbb{C}^{\mathbb{N}_0}$.

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Construction of Schwartz spaces E with only finite-dimensional bounded sets and $\Delta(E) \neq \mathbb{C}^{\mathbb{N}_0}$.

Theorem (2017, F. Bastin, L.D.)

There exists a family of Schwartz (or even nuclear) non-metrizable spaces E for which

$$\Delta(E) \subsetneq \Delta_b(E).$$

Thank you for your attention!

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