Non-separable Banach spaces studied via projectional skeletons and “rich” families of their separable subspaces

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Valencia, October 2017

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(iv) Given a sequence $s_1 < s_2 < \cdots$ in Γ, then $P_{\sup_{n\in\mathbb{N}} s_n} X = \overline{\bigcup_{n\in\mathbb{N}} P_{s_n} X}$.

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What does Γ usually look like?

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(ii) There exists a rich family $\mathcal{A} \subset \mathcal{S}_{\sqsubset}(X \times X^*)$ such that for every $V \times Y \in \mathcal{A}$ the assignment $Y \ni x^* \longmapsto x^*|_V \in V^*$ is a `surjection` *and an isometry as well.*

(iii) X^* admits a projectional skeleton $(P_s : \ s \in \Gamma)$ such that $\bigcup\{P_s{}^* X^{**} : \ s \in \Gamma\} \supset X$.

Proof. (i)⇒(ii) profits from a long bow of ideas across half a century: [Lindenstrauss65, Amir-Lindenstrauss68, Tacon70, Simons 1972, John-Zizler74, Gul'ko79, Jayne-Rogers85, Fabian-Godefroy88, Stegall94,Cúth-Fabian15].

(ii)⇒(iii) is very easy; just take $\Gamma := \mathcal{A}$.

(iii)⇒(i) is quite easy.

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e.g. $L_1(\mu)$, with `any` σ-additive measure μ, duals to C^* algebras, order continuous lattices, $C(G)$, with G a compact abelian group, and preduals of semifinite von Neumann algebras

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We have an elementary proof of the necessity (i.e., without using elementary submodels from logic).

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