DEPARTAMENTO DE ANÁLISE MATEMÁTICA, ESTATÍSTICA E OPTIMIZACIÓN

Variable exponent sequence spaces revisited

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H. Lebesgue (1875–1941)

Lebesgue norms

$$\|x\|_p = \left(\sum_{i=1}^{n} |x_i|^p \right)^{\frac{1}{p}}, \quad 1 \leq p < \infty.$$

$$\|x\|_\infty = \sup_{i \in \mathbb{N}} |x_i|$$

Lebesgue spaces

$$\ell^p = \{ x = (x_i)_{i \in \mathbb{N}} : \|x\|_p < \infty \}$$

$$\ell^\infty = \{ x = (x_i)_{i \in \mathbb{N}} : \|x\|_\infty < \infty \}$$

Hölder inequality

$$\sum_{i=1}^{\infty} |x_i \, y_i| \leq \|x\|_p \, \|y\|_{p^*}$$

H. Lebesgue (1875–1941)

Let X be a Banach space.

- If X has a normalized basis $(x_n)_{n\in\mathbb{N}}$ which is equivalent to all its normalized block basis then such basis is equivalent to the unit vector basis in c_0 or in some ℓ^p, $1 \leq p < \infty$.

Let $X = c_0$ or $X = \ell^p$, $1 \leq p \leq \infty$.

- Then every infite dimensional complemented subspace of X is isomorphic to X.

W. Orlicz (1903–1990)

Let $p\colon \mathbb{N} \longrightarrow (1,\infty)$ be fixed and $x = (x_i)_{i\in\mathbb{N}}$ a sequence of numbers such that

$$\sum_{i=1}^{n} |x_i|^{p(i)} < \infty.$$

Problem: to give a necesary and sufficient condition for $y = (y_i)_{i\in\mathbb{N}}$ to be such that

$$\sum_{i=1}^{\infty} |x_i\, y_i| < \infty.$$

Solution: it is necessary and sufficient that it exists $\lambda > 0$ satisfying

$$\sum_{i=1}^{\infty} |\lambda\, x_i|^{p_i^*} < \infty.$$

W. Orlicz (1903–1990)

Orlicz function

Given a function $\phi\colon [0,\infty) \longrightarrow [0,\infty)$ it is said to be an *Orlicz function* if:

- ϕ is continuous, increasing and convex
- $\phi(0) = 0$ and $\lim_{t\to\infty} \phi(t) = \infty$

Luxemburg-Nakano norms

$$\|x\|_\phi = \inf\left\{\lambda > 0 : \sum_{i=1}^{\infty} \phi\Big(\frac{|x_i|}{\lambda}\Big) \le 1\right\}$$

Orlicz spaces

$$\ell^\phi = \{x = (x_i)_{i\in\mathbb{N}} : \|x\|_\phi < \infty\}$$

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Luxemburg-Nakano norms

$$\|x\|_\Phi = \inf\left\{\lambda > 0 : \sum_{i=1}^{\infty} \phi_i\Big(\frac{|x_i|}{\lambda}\Big) \leq 1\right\}$$

Generalized Orlicz spaces

$$\ell^\Phi = \{x = (x_i)_{i\in\mathbb{N}} : \|x\|_\Phi < \infty\}$$

 W. Orlicz (1903–1990)

"Importance" of spaces ℓ^ϕ and ℓ^Φ

– *Why are Orlicz spaces "better" than Lebesgue ℓ^p spaces?*

– *Tell me first why Lebesgue ℓ^p spaces are "better" than ℓ^2?*

Usefulness of spaces ℓ^ϕ y ℓ^Φ

- Structure of ℓ^φ spaces is richer (but also more complicated!) than classical ℓ^p spaces
- Analysis of spaces $\ell^p \oplus \ell^r$ with $p \neq r$
- Subspaces of $\mathsf{L}^1(0,1)$
- Spaces of variable exponent

A different viewpoint: key idea or motivation

Given two real (or complex) **numbers** x, y:

$$x \boxplus_p y = \begin{cases} \Big[|x|^p + |y|^p\Big]^{1/p}, & 1 \le p < \infty; \\ \max\{|x|,\, |y|\}, & p = \infty. \end{cases}$$

A different viewpoint: key idea or motivation

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Given a **sequence** $x = (x_n)_{n\in\mathbb{N}}$ of real (or complex) numbers:

$$\begin{aligned} \|x\|_p &= \Big(\sum_{n=1}^{\infty} |x_n|^p\Big)^{\frac{1}{p}} \\ &= \Big(\sum_{n\in N_1}^{\infty} |x_n|^p\Big)^{\frac{1}{p}} \boxplus_p \Big(\sum_{n\in N_2}^{\infty} |x_n|^p\Big)^{\frac{1}{p}} \boxplus_p \Big(\sum_{n\in N_3}^{\infty} |x_n|^p\Big)^{\frac{1}{p}} \\ &= \|\mathbb{1}_{N_1} x\|_p \boxplus_p \|\mathbb{1}_{N_2} x\|_p \boxplus_p \|\mathbb{1}_{N_3} x\|_p \end{aligned}$$

A different viewpoint: key idea or motivation

Given two real (or complex) **numbers** x, y:

$$x \boxplus_p y = \begin{cases} \left[|x|^p + |y|^p\right]^{1/p}, & 1 \le p < \infty; \\ \max\{|x|, |y|\}, & p = \infty. \end{cases}$$

Given a measurable **function** f:

$$\begin{aligned} \|f\|_p &= \left(\int_\Omega |f(x)|^p \, dx \right)^{\frac{1}{p}} \\ &= \left(\int_{E_1} |f(x)|^p \, dx \right)^{\frac{1}{p}} \boxplus_p \left(\int_{E_2} |f(x)|^p \, dx \right)^{\frac{1}{p}} \\ &= \|\mathbb{1}_{E_1} f\|_p \boxplus_p \|\mathbb{1}_{E_2} f\|_p \end{aligned}$$

A different viewpoint: the rigorous definition

Let $p\colon \mathbb{N} \longrightarrow [1,\infty]$ be a map and
$x = (x_n)_{n\in\mathbb{N}} \in \ell^\infty$ be a sequence of real (or complex) numbers.

We define:

$$\|x\|_{(1)} = x_1 \boxplus_{p_1} x_2 = \Big[|x_1|^{p_1} + |x_2|^{p_1}\Big]^{1/p_1}$$

$$\|x\|_{(k)} = \|x\|_{(k-1)} \boxplus_{p_{k-1}} x_k = \Big[\|x\|_{(k-1)}^{\,p_{k-1}} + |x_k|^{p_{k-1}}\Big]^{1/p_{k-1}}, \quad k \geq 2$$

We have:

- $\|\cdot\|_{(k)}$ is a seminorm in ℓ^∞ for each $k \in \mathbb{N}$
- $(\|x\|_{(k)})_{k\in\mathbb{N}}$ is a non-decreasing sequence
- The following map is well defined

$$\boldsymbol{\Phi}\colon x \in \ell^\infty \longmapsto \boldsymbol{\Phi}(x) = \lim_{k\to\infty} \|x\|_{(k)} \in [0,\infty]$$

A different viewpoint: the rigorous definition

Let $p\colon \mathbb{N} \longrightarrow [1,\infty]$ be a map and
$x = (x_n)_{n\in\mathbb{N}} \in \ell^\infty$ be a sequence of real (or complex) numbers.

We define:

$$\boldsymbol{\ell^{p(\cdot)} = \{x = (x_n)_{n\in\mathbb{N}} \in \ell^\infty \text{ such that } \Phi(x) < \infty\}}$$

We have:

- $\ell^{p(\cdot)}$ is a vector space
- $\|x\|_{\ell^{p(\cdot)}} = \Phi(x)$ defines a norm in $\ell^{p(\cdot)}$
- $\|P_i(x)\| \leq \|P_j(x)\|$ if $i \leq j$ and

$$\|x\|_{\ell^{p(\cdot)}} = \sup_{k\in\mathbb{N}} \|P_k(x)\|_{\ell^{p(\cdot)}} = \sup_{k\in\mathbb{N}} \|x\|_{(k)}, \quad x \in \ell^{p(\cdot)}$$

A different viewpoint: "importance" of $\ell^{p(\cdot)}$

Theorem: There exists a Banach space -of type $\ell^{p(\cdot)}$- such that:

- **it has an 1-unconditional Schauder basis**
- **it contains all spaces ℓ^p, $1 \leq p < \infty$, isomorphically (almost isometrically)**

Spaces X and Y are almost isometric if $d_{\mathrm{BM}}(X,Y) = 1$, where

$$d_{\mathrm{BM}}(X,Y) = \inf\left\{ \|T\|\,\|T^{-1}\| \text{ such that } T\colon X \longrightarrow Y \text{ isomorphism} \right\}$$

X is contained almost isometrically in Z if for each $\varepsilon > 0$ there exists a subspace $Y \subset Z$ such that $d_{\mathrm{BM}}(X,Y) < 1 + \varepsilon$.

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Pitt's theorem ℓ^p and ℓ^q are not isomorphic when $p \neq q$, $1 \leq p, q < \infty$.

!!! $\mathcal{C}[0,1]$ is universal for all separable Banach spaces, but it does not admit any unconditional basis!

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- **it contains all spaces ℓ^p, $1 \leq p < \infty$, isomorphically (almost isometrically)**

Proof: enumerate $\mathbb{Q} \cap [1, \infty) = \{q(n) : n \in \mathbb{N}\}$ and consider $\ell^{q(\cdot)}$

Lemma 1: If $(n_k)_{k \in \mathbb{N}} \subset \mathbb{N}$ is a sequence, then

$$\{x = (x_j)_{j \in \mathbb{N}} : \text{supp}(x) \subset \{n_1, n_1 + 1, n_2 + 1, n_3 + 1, \dots\}\}$$

is isometric to $\ell^{q(\cdot)}$, where $q(k) = p(n_k)$ for each $k \in \mathbb{N}$.

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Proof: enumerate $\mathbb{Q} \cap [1, \infty) = \{q(n) : n \in \mathbb{N}\}$ and consider $\ell^{q(\cdot)}$

Lemma 2: If $\psi_j = \mathrm{Id} : \ell^{p|^{(j+1)}q} \longrightarrow \ell^{p|^{(j)}q}$ then

$$\|\psi_j\| = \|\mathrm{Id} : \ell_2^{p_j} \longrightarrow \ell_2^{q_j}\|$$

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- **it contains all spaces ℓ^p, $1 \leq p < \infty$, isomorphically (almost isometrically)**

Proof: enumerate $\mathbb{Q} \cap [1, \infty) = \{q(n) : n \in \mathbb{N}\}$ and consider $\ell^{q(\cdot)}$

Lemma 3: Let $p, q : \mathbb{N} \longrightarrow [1, \infty]$ and $\varepsilon > 0$. If

$$\liminf_{n \to \infty} |p(n) - q(k)| = 0, \quad \text{for all } k \in \mathbb{N}$$

then there is a strictly increasing sequence $(n_k)_{k \in \mathbb{N}} \subset \mathbb{N}$ such that ϕ defines an embedding $\ell^{q(\cdot)} \hookrightarrow \ell^{p(\cdot)}$ satisfying

$$(1 + \varepsilon)^{-1} \|y\| \leq \|\phi(y)\| \leq (1 + \varepsilon) \|y\|$$

A different viewpoint: "importance" of $\ell^{p(\cdot)}$

Theorem: There exists a Banach space -of type $\ell^{p(\cdot)}$- such that:

- **it has an 1-unconditional Schauder basis**
- **it contains all spaces ℓ^p, $1\leq p<\infty$, isomorphically (almost isometrically)**

Proof: enumerate $\mathbb{Q}\cap[1,\infty)=\{q(n): n\in\mathbb{N}\}$ and consider $\ell^{q(\cdot)}$

More exactly, consider $[(e_n)_{n\in\mathbb{N}}]\subset\ell^{q(\cdot)}$!

But, what is exactly $[(e_n)_{\mathbb{N}}]$?? We only know that $[(e_n)_{\mathbb{N}}]\subset\ell^{q(\cdot)}\cap c_0$!

J. Talponen, *A natural class of sequential Banach spaces*,
Bull. Polish Acad. of Sciences Mathematics 59:2 (2011), 186–196.

A different viewpoint: comparisson with previous ideas

Let $p\colon \mathbb{N} \longrightarrow [1,\infty]$ be a sequence and $p^*\colon \mathbb{N} \longrightarrow [1,\infty]$ such that

$$1/p_n + 1/p_n^* = 1, \quad n \in \mathbb{N}.$$

- If $p(n) = p$ for all $n \in \mathbb{N}$ then $\ell^{p(\cdot)} = \ell^p$
- Hölder inequality

$$\sum_{n=1}^{\infty} |x_n||y_n| \leq 1 \|x\|_{\ell^{p(\cdot)}} \|y\|_{\ell^{p^*(\cdot)}}$$

- Norm computation and permutations

$$\|x_n\|_{(M_n)} = \|x_{\pi(n)}\|_{M_{\pi(n)}}$$

$$\|(1,1,1)\|_{[\mathbb{R}\oplus_1\mathbb{R}]\oplus_2\mathbb{R}} = \sqrt{5} \neq \sqrt{2} + 1 = \|(1,1,1)\|_{[\mathbb{R}\oplus_2\mathbb{R}]\oplus_1\mathbb{R}}$$

$[(e_n)_{\mathbb{N}}] = \ell^{q(\cdot)} \cap c_0$

Banach Function Spaces

Definition: A given map $\rho\colon \mathcal{M}^+ \longrightarrow [0,\infty]$ it is said to be a **Banach norm** if:

$$\rho(f) = 0 \Leftrightarrow f = 0 \;\; \mu\text{-a.e.},$$

$$\rho(af) = a\rho(f) \quad \forall\, f \in \mathcal{M}^+,\ \forall\, a \geq 0,$$

$$\rho(f+g) \leq \rho(f) + \rho(g) \quad \forall\, f, g \in \mathcal{M}^+;$$

$$0 \leq g \leq f \;\; \mu\text{-a.e.} \Rightarrow \rho(g) \leq \rho(f) \quad \forall\, f, g \in \mathcal{M}^+;$$

$$0 \leq f_n \uparrow f \;\; \mu\text{-a.e.} \Rightarrow \rho(f_n) \uparrow \rho(f) \quad \forall\, f \in \mathcal{M}^+,\ \forall\, (f_n)_{n\in\mathbb{N}} \subset \mathcal{M}^+;$$

$$\mu(E) < \infty \Rightarrow \rho(\chi_E) < \infty \quad \text{for all } \mu\text{-measurable set } E \subset R;$$

if $E \subset R$ is a μ-measurable set such that $\mu(E) < \infty$ then

$$\int_E f(x)\, d\mu(x) \leq C_E\, \rho(f), \quad \forall\, f \in \mathcal{M}^+.$$

$[(e_n)_{\mathbb{N}}] = \ell^{q(\cdot)} \cap c_0$

Definition

Given a Banach Function Space X, it is said that a function $f \in X$ has absolutely continuous norm in X if $\| f\chi_{E_n} \| \to 0$ for every sequence $(E_n)_{n\in\mathbb{N}}$ of μ-measurable sets such that $E_n \to \emptyset$ μ-a.e.

Definition

Let X be a Banach Function Space. The closure in X of the set formed by the simple functions is denoted by X_b.

$[(e_n)_{\mathbb{N}}] = \ell^{q(\cdot)} \cap c_0$

Theorem 1 Space $\ell^{p(\cdot)}$ is an example of Banach Function Space.

Theorem 2 The associated space of $\ell^{p(\cdot)}$, which wil be denoted by $[\ell^{p(\cdot)}]'$, coincides with Talponen space $\ell^{p^*(\cdot)}$.

Theorem 3 The space formed by the sequences of $\ell^{p(\cdot)}$ with absolutely continuous norm, which wil be denoted by $[\ell^{p(\cdot)}]_a$, coincides with $\ell^{p(\cdot)} \cap c_0$.

Proposition 4 Moreover, it is clear that $\ell_b^{p(\cdot)} = [e_n : n \in \mathbb{N}]$.

$[(e_n)_{\mathbb{N}}] = \ell^{q(\cdot)} \cap c_0$

The inclusion $X_a \subset X_b$ is valid for any Banach Function Space X. In the case of Talponen type spaces, such result is translated into the inclusion

$$\ell^{p(\cdot)} \cap c_0 \subset [e_n : n \in \mathbb{N}].$$

Since the opposite inclusion is also true, we conclude that for any sequence $p \colon \mathbb{N} \longrightarrow [1, \infty]$,

$$\ell_a^{p(\cdot)} = \ell^{p(\cdot)} \cap c_0 = [e_n : n \in \mathbb{N}] = \ell_b^{p(\cdot)}.$$

Such result about Talponen spaces coincides with the fact that in a Banach Function Space X the subspaces X_a and X_b coincide if and only if the characteristic function χ_E has absolutely continuous norm for every measurable set E with finite measure.

Some questions I

$$
\begin{aligned}
\|x\| &= \Big(\sum_{n=1}^{\infty}|x_n|^p\Big)^{\frac{1}{p}} \\
&= \Big(\sum_{n\in N_1}^{\infty}|x_n|^p\Big)^{\frac{1}{p}} \boxplus_p \Big(\sum_{n\in N_2}^{\infty}|x_n|^p\Big)^{\frac{1}{p}} \boxplus_p \Big(\sum_{n\in N_3}^{\infty}|x_n|^p\Big)^{\frac{1}{p}} \\
&= \|\mathbb{1}_{N_1}x\|_p \boxplus_p \|\mathbb{1}_{N_2}x\|_p \boxplus_p \|\mathbb{1}_{N_3}x\|_p
\end{aligned}
$$

$$
\begin{aligned}
\|f\| &= \Big(\int_{\Omega}|f(x)|^p\,dx\Big)^{\frac{1}{p}} \\
&= \Big(\int_{E_1}|f(x)|^p\,dx\Big)^{\frac{1}{p}} \boxplus_p \Big(\int_{E_2}|f(x)|^p\,dx\Big)^{\frac{1}{p}} \boxplus_p \Big(\int_{E_3}|f(x)|^p\,dx\Big)^{\frac{1}{p}} \\
&= \|\mathbb{1}_{E_1}f\|_p \boxplus_p \|\mathbb{1}_{E_2}f\|_p \boxplus_p \|\mathbb{1}_{E_3}f\|_p
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$$

Some questions I

$$
\begin{aligned}
\|x\| &= \Big(\sum_{n=1}^{\infty} |x_n|^p\Big)^{\frac{1}{p}} \\
&= \left[\Big(\sum_{n\in N_1}^{\infty} |x_n|^p\Big)^{\frac{1}{p}} \boxplus_p \ \Big(\sum_{n\in N_2}^{\infty} |x_n|^p\Big)^{\frac{1}{p}}\right] \boxplus_p \Big(\sum_{n\in N_3}^{\infty} |x_n|^p\Big)^{\frac{1}{p}} \\
&= \left[\|\mathbb{1}_{N_1} x\|_p \ \boxplus_p \ \|\mathbb{1}_{N_2} x\|_p\right] \boxplus_p \|\mathbb{1}_{N_3} x\|_p
\end{aligned}
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&= \left[\|\mathbb{1}_{E_1} f\|_p \ \boxplus_p \ \|\mathbb{1}_{E_2} f\|_p\right] \boxplus_p \|\mathbb{1}_{E_3} f\|_p
\end{aligned}
$$

Some questions I

$$\|x\| = \Big(\sum_{n=1}^{\infty} |x_n|^p\Big)^{\frac{1}{p}}$$

$$= \Big(\sum_{n\in N_1}^{\infty} |x_n|^p\Big)^{\frac{1}{p}} \boxplus_p \left[\Big(\sum_{n\in N_2}^{\infty} |x_n|^p\Big)^{\frac{1}{p}} \boxplus_p \Big(\sum_{n\in N_3}^{\infty} |x_n|^p\Big)^{\frac{1}{p}}\right]$$

$$= \|\mathbb{1}_{N_1} x\|_p \boxplus_p \left[\|\mathbb{1}_{N_2} x\|_p \boxplus_p \|\mathbb{1}_{N_3} x\|_p\right]$$

$$\|f\| = \Big(\int_\Omega |f(x)|^p \, dx\Big)^{\frac{1}{p}}$$

$$= \Big(\int_{E_1} |f(x)|^p \, dx\Big)^{\frac{1}{p}} \boxplus_p \left[\Big(\int_{E_2} |f(x)|^p \, dx\Big)^{\frac{1}{p}} \boxplus_p \Big(\int_{E_3} |f(x)|^p \, dx\Big)^{\frac{1}{p}}\right]$$

$$= \|\mathbb{1}_{E_1} f\|_p \boxplus_p \left[\|\mathbb{1}_{E_2} f\|_p \boxplus_p \|\mathbb{1}_{E_3} f\|_p\right]$$

Some questions I

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\begin{aligned}
\|x\| &= \Big(\sum_{n=1}^{\infty} |x_n|^p\Big)^{\frac{1}{p}} \\
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&= \|\mathbb{1}_{N_1} x\|_p \boxplus_p \|\mathbb{1}_{N_2} x\|_p \boxplus_p \|\mathbb{1}_{N_3} x\|_p
\end{aligned}
$$

Properties of $\boxplus_p$

- $x \boxplus_p y = y \boxplus_p x$
- $(x \boxplus_p y) \boxplus_p z = x \boxplus_p (y \boxplus_p z)$
- $(x \boxplus_p y) \boxplus_q z \neq x \boxplus_p (y \boxplus_q z)$

Some questions II

Schur property

We say that X has Schurs's property if $(x_n)_{n\in\mathbb{N}}$ converging weakly to $x \in X$ implies that $\|x_n - x\| \to 0$ as $n \to \infty$.

In other words, the weak and strong topologies share the same convergent sequences.

Theorem: If $p\colon \mathbb{N} \longrightarrow [1,\infty]$ is a given sequence such that

$$\lim_{n\to\infty} p_n = 1$$

then $\ell^{p(\cdot)}$ has the Schur property.

THANK YOU FOR YOUR ATTENTION

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