Interpolating sequences for weighted spaces of analytic functions on the unit ball of a Hilbert space

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- $S_w := (\varphi_w(z_j))$ is the image of the sequence (z_j) under the automorphism $\varphi_w : \mathbf{D} \to \mathbf{D}$ given by

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- (z_n) is linear interpolating if the map S has a linear right inverse.

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Necessary conditions for interpolation

Theorem. Let υ be a weight and $F : [0,1) \to \mathbb{R}^+$ be a continuous non decreasing function s.t.

$$\frac{\upsilon(x)}{\upsilon(y)} \leq F(\rho_E(x, y)), \quad x, y \in B_E. \tag{1.1}$$

Then any interpolating sequence (w_n) for $H^\infty_\upsilon(B_E)$ is hyperbolically separated.

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Remark. The standard weights $\upsilon_\alpha(x) = (1 - \|x\|^2)^\alpha$, for $\alpha \geq 0$, satisfy the assumption with $F(r) = \left(\frac{4}{1-r^2}\right)^\alpha$.

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- In the infinite dimensional case (for instance $E=\ell_2$):

  Consider u a bounded radial weight on $\mathbf{D}$ and $v(x) = u(\|x\|)$ for $x \in B_E$. Consider an orthonormal sequence $(e_n) \subset E$ and $(z_n) \subset \mathbf{D}$ such that $(\inf_n |z_n|)(\inf_n u(|z_n|)) > 0$.

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  Then the sequence $(z_n e_n)$ is interpolating for for $H^\infty_v(B_E)$ by the $d-$homogeneous polynomial

$$P_d(x) = \sum_{n=1}^{\infty} \frac{\alpha_n}{u(|z_n|)z_n^d} \langle x, e_n \rangle^d.$$

  If $\sup_n |z_n| < 1$, then $\lim_n \|z_n e_n\| \neq 1$.

Interpolating sequences. Sufficient conditions

(Easy) sufficient conditions for a sequence to be interpolating for $H^\infty_{\upsilon_\alpha}(B_E)$

Remark [Lindström, Galindo, M'09]. If the sequence $(w_n) \subset B_E$ is interpolating for $H^\infty(B_E)$, then it is also linear interpolating.

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3) If $\lim_{n\to\infty} \|w_n\| = 1$, there exists a subsequence (w_{n_k}) which is linear interpolating.

Sufficient conditions

Carleson measures on the unit ball B_E (for E finite or infinite dimensional)

For any $\xi \in E$, $\|\xi\| = 1$ and $0 < h < 1$ the *Carleson window* $S(\xi, h)$ is given by

$$S(\xi, h) = \{y \in B_E : |1 - \langle y, \xi \rangle| < 2h\}.$$

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Definition. Let η be a finite Borel measure on B_E and $\beta > 0$. We say that η is a β-Carleson measure whenever there exists $C > 0$ such that

$$\eta(S(\xi, h)) \leq Ch^{\beta}, \qquad \text{for all } \|\xi\| = 1 \text{ and all } 0 < h < 1.$$

We write $\|\eta\|_{\beta} = \sup \left\{ \frac{\eta(S(\xi,h))}{h^{\beta}} : \|\xi\| = 1,\ 0 < h < 1 \right\}.$

Carleson measures II

Lemma. Let η be a finite Borel measure on B_E and $\beta > 0$. Define for $\alpha > 0$

$$I_\eta(\alpha, \beta) = \sup_{\|x\|<1} \frac{1}{(1 - \|x\|^2)^\beta} \int_{B_E} \left(\frac{|1 - \|x\|^2|}{|1 - \langle w, x \rangle|} \right)^\alpha d\eta(w) \in [0, \infty].$$

(i) If $I_\eta(\alpha, \beta) < \infty$ for some $\alpha > 0$, then η is a β-Carleson measure.

(ii) If η is a β-Carleson measure, then $I_\eta(\alpha, \beta) < \infty$ for any $\alpha > \beta$.

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Definition. Given $(w_n)_{n=1}^\infty \subset B_E$ and $\gamma > 0$ we define

$$\eta_{\gamma,(w_n)} = \sum_{n=1}^{\infty} (1-\|w_n\|^2)^\gamma \delta_{w_n}. \qquad (1.2)$$

In particular $\eta_{\gamma,(w_n)}(B_E) < \infty$ if and only if $\sum_{n=1}^\infty (1-\|w_n\|^2)^\gamma < \infty$.

Interpolating sequences in B_n

Proposition [Massaneda'95] Let $E = \mathbb{C}^n$ and $\beta \geq n$. We have that

$\eta_{\beta,(z_j)}$ is a β-Carleson measure if and only if $K(\{z_j\}, \alpha, \beta) < \infty$ for any $0 < \alpha \leq \beta$.

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For $\alpha, \beta > 0$ and $(w_j) \subset B_E$, we denote

$$K(\{w_j\}, \alpha, \beta) = \sup_{k \in \mathbb{N}} \sum_{j \neq k} \frac{(1 - \|w_k\|^2)^\alpha (1 - \|w_j\|^2)^\beta}{|1 - \langle w_k, w_j \rangle|^{\alpha+\beta}}.$$

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Remark. For any $x, y \in B_E$ we have

$$1 - \rho_E(x, y)^2 = \frac{(1 - \|x\|^2)(1 - \|y\|^2)}{|1 - \langle x, y \rangle|^2}.$$

Interpolating sequences in B_n

Theorem [Massaneda '95]. Let $\alpha > 0$ and $(z_n) \subset \mathbb{B}_n$.

(i) If (z_n) is interpolating for $H^\infty_{v_\alpha}(\mathbb{B}_n)$, then

- $K(\{z_j\}, \alpha, \beta) < \infty \ \forall \beta \geq \alpha$ with $\beta > n$
- (and (z_n) is hyperbolically separated).

(ii) If $\exists \beta \geq \max\{n, \alpha\}$ with $K(\{z_j\}, \alpha, \beta) < 1$, then (z_j) is interpolating for $H^\infty_{v_\alpha}(\mathbb{B}_n)$.

Interpolating sequences in B_n

Theorem [Massaneda '95]. Let $\alpha > 0$ and $(z_n) \subset \mathbb{B}_n$.

(i) If (z_n) is interpolating for $H^\infty_{v_\alpha}(\mathbb{B}_n)$, then

- $K(\{z_j\}, \alpha, \beta) < \infty \ \forall \beta \geq \alpha$ with $\beta > n$
- (and (z_n) is hyperbolically separated).

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So...

- If (z_n) interpolating for $H^\infty_{v_\alpha}(\mathbb{B}_n) \longrightarrow \eta_{\beta,(z_j)}$ is $\beta-$Carleson for $\beta \geq \alpha$, $\beta > n$.
- If $\exists \beta \geq n, \alpha \ : \ \left.\begin{array}{l} \eta_{\beta,(z_j)} \text{ Carleson} \\ K(\{z_j\}, \alpha, \beta) < 1 \end{array}\right\} \longrightarrow (z_n)$ is interpolating for $H^\infty_{v_\alpha}(\mathbb{B}_n)$.

Results

Remark. Replace $\mathbb{B}_n$ by B_E $\longrightarrow$ there exist interpolating sequences in $H^\infty_{v_\alpha}(B_E)$ which may have $K(\{w_j\}, \alpha, \beta) = \infty$ for any $\beta > 0$.

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For instance, the sequence $(z_n e_n)$. Considering u a bounded radial weight on $\mathbf{D}$ and $v(x) = u(\|x\|)$ for $x \in B_E$. Also an orthonormal sequence $(e_n) \subset E$ and $(z_n) \subset \mathbf{D}$ such that $(\inf_n |z_n|)(\inf_n u(|z_n|)) > 0$. Take (z_n) such that $\sup |z_n| < 1$.

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In $E=\ell_2$, use $w_j=\frac{1}{2}e_j$. Then $\rho_E(w_j,w_k)=\frac{\sqrt{7}}{4}$ for $k\neq j$ and $\sum_j(1-\|w_j\|^2)^\beta=\infty$.

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then it is linear interpolating for $H^\infty_{v_\alpha}(B_E)$.

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Sketch of the proof. Consider the linear operator $S : H^\infty_{v_\alpha}(B_E) \to \ell_\infty$ given by

$$S(f) = \Big((1 - \|w_n\|^2)^\alpha f(w_n) \Big)_n .$$

We will find a linear bounded operator $\Phi : \ell_\infty \to H^\infty_{v_\alpha}(B_E)$ such that $\|Id - S \circ \Phi\| < 1$.

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- The Bloch space $\mathcal{B}(B_E)$ is the set of analytic functions $f : B_E \to \mathbb{C}$:

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Interpolating sequences for $\mathcal{B}(B_E)$

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$$\text{For any } \alpha = (\alpha_n) \in \ell_\infty \longrightarrow \exists f \in \mathcal{B}(B_E) : (1 - \|x_n\|^2) Rf(x_n) = \alpha_n.$$

- $\mathcal{B}^\alpha(B_E)$ for any $\alpha > 0$ if we change the weight $(1 - \|z\|^2)$ by $(1 - \|z\|^2)^\alpha$.

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(iii) If $\sum_{n=1}^\infty (1 - \|w_n\|^2)^2 \delta_{w_n}$ is a 2-Carleson measure and (w_n) is hyperb. separated for some $R > \sqrt{1 - \left(\frac{2^{\alpha/2} - 1}{2^{\alpha+2}\|\eta_{2,(w_n)}\|_2}\right)^{2/\alpha}}$, it is linear interpolating for $\mathcal{B}^\alpha(B_E)$.

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There exist sequences $(x_n) \subset B_E$ which are interpolating for $\mathcal{B}(B_E)$ but not interpolating for $H^\infty(B_E)$.

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Consider k an even number, $k \geq 2$ and circles C_n centered at 0 and radius $r_n = 1 - \frac{1}{k^n}$ for any $n \geq 1$. In each circle C_n, we take $z_{n,j} = r_n e^{\frac{2\pi i j}{k^{n-1}}}$ for any $0 \leq j < k^{n-1}$.

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There exist sequences $(x_n) \subset B_E$ which are interpolating for $\mathcal{B}(B_E)$ but not interpolating for $H^\infty(B_E)$.

Sketch of the proof. There exists sequences which are hyperbolically separated for R so close to 1 as we want but $\sum_z (1 - \|z\|^2) = \infty$.

Consider k an even number, $k \geq 2$ and circles C_n centered at 0 and radius $r_n = 1 - \frac{1}{k^n}$ for any $n \geq 1$. In each circle C_n, we take $z_{n,j} = r_n e^{\frac{2\pi ij}{k^{n-1}}}$ for any $0 \leq j < k^{n-1}$.

For z, w in the sequence,

$$\rho(z,w) \geq \min \left\{ \frac{k-1}{k+1}, \frac{1}{\sqrt{1 + \left(\frac{2k^n - 1}{4k(k^n - 1)}\right)^2}} \right\} \to 1 \quad \text{when } k \to \infty.$$

$$\sum_{n=1}^{\infty} (1 - |z_n|) = \sum_{k=1}^{\infty} k^{n-1}(1 - r_n) = \sum_{k=1}^{\infty} k^{n-1} \frac{1}{k^n} = \sum_{k=1}^{\infty} \frac{1}{k} = \infty.$$

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Thanks for your attention!