Pontryagin maximum principle for optimal nonpermanent control problems on time scales.

Loïc Bourdin (University of Limoges, France)

Emmanuel Trélat (University of Paris 6, France)

Pontryagin maximum principle (in short, PMP) is a fundamental result of the optimal control theory. In its classical statement, the control of the dynamical system is considered as *permanent*, that is, the control is authorized to be modified at any real time. In many problems, it follows that achieving the optimal trajectory requires a permanent modification of the control. However, such a request is not conceivable in practice for human beings, even for mechanical or numerical devices. Therefore, piecewise constant controls (also called *sampled-data controls* or *digital controls*), for which only a finite number of modifications is authorized, are usually considered in Automatic and Engineering. Sampled-data controls are one example of *nonpermanent controls*. Another example concerns dynamical systems whose trajectories go across noncontrolled areas (like a mobile phone or a GPS device going under a tunnel).

In this talk, we will present a new version of the PMP that can be applied to optimal nonpermanent control problems. This result was recently obtained in [1] (see also [2]) and is stated with the help of the *time scale calculus* theory. Numerous properties about optimal permanent controls are well-known in the literature (such as the continuity of the Hamiltonian, or the saturation of the control constraints set when the Hamiltonian is affine, etc.). In this talk, we will be interested in the preservation (or not) of these classical properties when we consider nonpermanent controls. Finally, in the linear-quadratic case, we will state that the optimal sampled-data controls converge to the optimal permanent control when the distances between consecutive sampling times uniformly tend to zero.

References

[1] L. Bourdin and E. Trélat. Optimal sampled-data control, and generalizations on time scales. *Mathematical Control and Related Fields*, 6(1):53-94, 2016.

[2] L. Bourdin and E. Trélat. Pontryagin maximum principle for optimal sampled-data control problems. *In proceedings of 16th IFAC workshop on CAO*, 2015.