MATHEMATICAL TAPAS...

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This talk consists in presenting and commenting ten proposals taken from the book entitled *Mathematical tapas*, to appear in October 2016. Below is the foreword of this book.

Mathematical tapas... but what are tapas? *Tapas* is a Spanish word (in the Basque country, on would also say *pintxos*) for small savory dishes typically served in bars, with drinks, shared with friends in a relaxed ambiance. The offer is varied, it may be meat, fish, vegetables,... Each guest of the party takes with a stick the tapas he likes best at the moment. This is the spirit of the mathematical tapas that we offer to the reader here.

Before explaining more precisely what these tapas are, let us briefly say what they are not. They are not, at least for the most part of them, questions extracted from competitions like mathematical olympiads, or national competitions for the best highschool students, or other sources of that kind. The mathematical questions posed at these events favour elementary number theory, inequalities on real numbers, plane geometry (especially properties of triangles). We have however selected some tasty tapas of this kind.

The tapas that we offer are mathematical questions to answer, exercises (more than long problems, in spirit). They concern **mathematics at the undergraduate level** (roughly speaking, from the end of highschool to the end of the first three years of universities[1]); they do not cover the whole spectrum of mathematics, of course. Clearly, they reflect the mathematical interests of the author:

Real analysis, Calculus:
functions of a real variable, multivariate calculus, differential calculus;
Matrices (especially positive semidefinite ones);
Convexity (sets and functions);
Optimization or "variational" situations...

As a general rule, the topics covered by the book are normally taught at most standard higher education institutions around the world.

How have they been chosen?

- Firstly, because "we like them". In other words, each tapa *shows or proves something*: it could be an interesting inequality among integers, on real numbers or integrals, a useful or surprising property of some mathematical objects, or simply an elegant formula... We are just sensitive to the aesthetics of mathematics.

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[1] "Licence" in the European Higher Education sytem.

- Secondly, because they illustrate the following motto: *"if you solve it, you learn something"*. During our career, we taught hundreds of students and, therefore, posed thousands of exercises (in directed sessions of exercise solving, for exams, etc.); but we have not kept here (standard) questions whose objective is just to test ability in calculating a derivative, an integral, eigenvalues, etc. We therefore have limited our choice of tapas for this first volume to the (symbolic) number of **333**.

Where have they been chosen?

I always had a soft spot for the "little" questions of mathematics, the ones that are asked among colleagues at the coffee machine or around a blackboard, in the university dining hall after lunch, and so on. Our mathematical tapas are chosen among them, as also in my favorite journals posing such challenges: the *American Mathematical Monthly*, the French mathematical journals entitled *Revue de la Filière Mathématique* (formerly *Revue de Mathématiques Spéciales*) and *Quadrature*. From time to time, I posed or solved questions posted in these journals. However, for many tapas, I must confess that I have forgotten their origin or history.

How are they classified?

As in restaurant guides, each tapa has one, two or three stars (★):

- One star (★). Tapas of the first level, for students at the end of the highschool or in the first year of university.

- Two stars (★★). Tapas of a more advanced level, more appropriate for students in the second year of university. That does not mean that solving them necessarily requires more expertise or wit than the one-starred ones, but sometimes just more mathematical maturity.

- Three stars (★★★). The upper level in the proposed tapas, typically for students in the third year of university. Some may be tough and need more chewing. This category contains our favorite tapas.

How are they presented?

Each tapa begins with a *statement*, of course. The statement may contain the answers to the posed questions; this is the case when the questions or proposals are of the form "Show that... " or "Prove that".

There are no detailed solutions to questions, that would have inflated this book by a factor three or four. Moreover, in mathematics, there is no uniform and unique way to write down answers. But, to help solving the posed challenges, we have proposed *hints*... They suggest a path to follow. A question without any indication could be impossible or too time-consuming... ; the same question with "spoon-fed" steps could be considered too easy. We have tried to keep an equilibrium between the two styles. Of course, an interested reader is encouraged to try to chew and swallow the tapas without having recourse to the hints.

When they are not integrated into the statements, we provide *answers* to questions, numerical results for example.

From time to time, we add some *comments*: on the context of the question, on its origin, on a possible extension.

An essential characteristic of mathematics is to be universal and thus international. So, imagine a student or someone who has some basic knowledge in mathematics in seclusion for some time on an isolated island, or just put into jail... With a pocket book like the one containing these tapas, he might pass an easier time and savour some of them.

Buen provecho!