The NSLUC property and Klee envelope

A. Jourani[1] [2] [3]
Université de Bourgogne
Institut de Mathématiques de Bourgogne
UMR 5584 CNRS
BP 47 870, 21078 Dijon Cedex, France
jourani@u-bourgogne.fr

Abstract

A notion called norm subdifferential local uniform convexity (NSLUC) is introduced and studied. It is shown that the property holds for all subsets of a Banach space whenever the norm is either locally uniformly convex or k-fully convex. The property is also valid for all subsets of the Banach space if the norm is Kadec-Klee and its dual norm is Gâteaux differentiable off zero. The NSLUC concept allows us to obtain new properties of the Klee envelope, for example a connection between attainment sets of the Klee envelope of a function and its convex hull. It is also proved that the Klee envelope with unit power plus an appropriate distance function is equal to some constant on an open convex subset as large as we need. As a consequence of obtained results, the subdifferential properties of the Klee envelope can be inherited from subdifferential properties of the opposite of the distance function to the complement of the bounded convex open set. Moreover the problem of singleton property of sets with unique farthest point is reduced to the problem of convexity of Chebyshev sets.

[1]L. Thibault, Université Montpellier II, Institut de Mathématiques, UMR 5149 CNRS, Case Courier 051, Place Eugène Bataillon 34095 Montpellier Cedex, France; and Centro de Modelamiento Matematico, Universidad de Chile, Chile. lionel.thibault@univ-montp2.fr

[2]D. Zagrodny, Cardinal Stefan Wyszynski University, Faculty of Mathematics and Natural Sciences, College of Science, Dewajtis 5, Warsaw, Poland. d.zagrodny@uksw.edu.pl