ON VECTOR MEASURES WITH SEPARABLE RANGE

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Abstract. Let X be a weakly Lindelöf determined Banach space. We prove that the following two statements are equivalent:

(i) Every Radon probability measure on (B_{X^*}, w^*) has separable support.

(ii) Every countably additive X^*-valued measure with σ-finite variation has norm separable range.

Some applications and related examples are given.

1. Introduction

In their pioneering work [4], Bartle, Dunford and Schwartz showed that the range of any countably additive measure with values in a Banach space is relatively weakly compact. In general, this is not true for norm compactness, even when we restrict our attention to indefinite Pettis integrals [9, 2D]. As regards vector measures with values in the dual X^* of a Banach space X, a result of Rybakov [17] (cf. [10, Corollary 10]) states that X does not contain subspaces isomorphic to ℓ^1 if and only if X^* has the so-called Compact Range Property, i.e. every countably additive X^*-valued measure with σ-finite variation has relatively norm compact range; in particular, such a vector measure has *norm separable* range.

Pettis integrable functions with norm separably-valued indefinite integral have been studied by several authors over the years, see [11, 15, 18]. It is worth pointing out that such functions can be approximated "weakly" by simple functions. More precisely, if f is a Pettis integrable function taking values in a Banach space Y and the indefinite integral of f has norm separable range, then there is a sequence (f_n) of Y-valued simple functions such that the family $\{\langle y^*, f_n\rangle : \; y^* \in B_{Y^*}, \; n \in \mathbb{N}\}$ is uniformly integrable and, for each $y^* \in Y^*$, we have $\lim_n \langle y^*, f_n\rangle = \langle y^*, f\rangle$ a.e., see [11, Theorem 3] or [18, 5-3-2]. For a detailed account on this topic we refer the reader to [12, 13]. The following question was posed by Musial in [12, Problem 22]: which Banach spaces Y have the *Pettis Separability Property*, that is, the indefinite integral of any Y-valued Pettis integrable function has norm separable range?

The aim of this paper is to discuss the norm separability of the range of a vector measure with values in the dual of a wide class of Banach spaces: those which are weakly Lindelöf determined (WLD for short, see below for the definition). Our main result (Theorem 2.3) states that *for a WLD Banach space X the following conditions are equivalent:*

(i) (B_{X^*}, w^*) *has the so-called property (M), i.e. every Radon probability measure on* (B_{X^*}, w^*) *has separable support.*

(ii) *Every countably additive X^*-valued measure with σ-finite variation has norm separable range.*

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We recall that (i) holds automatically whenever X is weakly compactly generated or, more generally, weakly $\mathcal{K}$-countably determined (every Gul'ko compact has property (M), see [2, Remarks 3.2] and [3]). Moreover, it is known that the validity of (i) for arbitrary WLD spaces is independent of ZFC, see [2, Remarks 3.2] and [16].

Some applications of Theorem 2.3 are also provided. On the one hand, we give a partial answer to the aforementioned Musial's question by showing that *X^* has the Pettis Separability Property whenever X is a WLD Banach space such that (B_{X^*}, w^*) has property (M)* (Corollary 2.5). On the other hand, it turns out that *if a WLD Banach space X does not contain subspaces isomorphic to ℓ^1, then (B_{X^*}, w^*) has property (M)* (Corollary 2.6).

We finish the paper by showing that for any Banach space X with a subspace isomorphic to $\ell^1(\omega_1)$, there exists a countably additive X^*-valued measure with finite variation whose range is not norm separable (Corollary 2.8).

Notation and terminology. As usual, we write ω_1 to denote the first uncountable ordinal. All unexplained notation and terminology can be found in our standard references [5] and [7].

Our Banach spaces $(Y, \|\cdot\|)$ are assumed to be real. By a 'subspace' of Y we mean a norm closed linear subspace. We write B_Y to denote the closed unit ball of Y and w^* stands for the weak* topology on Y^* (the topological dual of Y). Given $y^* \in Y^*$ and $y \in Y$, we write $\langle y^*, y\rangle$ to denote the evaluation of y^* at y.

Recall that Y is said to be *WLD* if (B_{Y^*}, w^*) is homeomorphic to some subset S of a cube $[-1,1]^\Lambda$, endowed with the product topology, such that for each $s \in S$ the set $\{\lambda \in \Lambda : s(\lambda) \neq 0\}$ is countable. The class of WLD Banach spaces contains all weakly compactly generated spaces and, more generally, all weakly $\mathcal{K}$-countably determined ones, see e.g. [8, Theorem 11.16] and [7, Theorem 7.2.7], respectively. As regards WLD spaces, we will only need Lemma 1.1 below, which follows from a standard argument used in the construction of projectional resolutions of the identity in non separable WLD spaces, see e.g. [7, Chapters 6 and 8]. We sketch a proof for the convenience of the reader.

Lemma 1.1. *Let Y be a WLD Banach space and S a w^*-separable subset of Y^*. Then there exist two subspaces $Y_0, Y_1 \subset Y$ such that $Y = Y_0 \oplus Y_1$, Y_0 is separable and $\langle y^*, y\rangle = 0$ for every $y^* \in S$ and every $y \in Y_1$. In particular, every bounded subset of S is w^*-metrizable.*

Sketch of proof. We can suppose without loss of generality that S is countable. Since Y is WLD, there is a set-valued mapping $\Phi : Y^* \longrightarrow 2^Y$ with the following properties:

- $\Phi(y^*)$ is countable for every $y^* \in Y^*$,
- $\{y^* \in \overline{B}^{w^*} : \langle y^*, y\rangle = 0 \text{ for every } y \in \Phi(B)\} = \{0\}$ for each non-empty set $B \subset Y^*$ for which $\overline{B}^{\|\cdot\|}$ is a subspace of Y^*,

see e.g. [7, Proposition 8.3.1]. (The couple (Y^*, Φ) is called a *projectional generator* on Y.) Fix a set-valued mapping $\Psi : Y \longrightarrow 2^{Y^*}$ such that, for each $y \in Y$, the set $\Psi(y) \subset B_{Y^*}$ is countable and $\|y\| = \sup\{\langle y^*, y\rangle : y^* \in \Psi(y)\}$. By [7, Lemma 6.1.3] we can find countable sets $A \subset Y$ and $S \subset B \subset Y^*$ such that $\overline{A}^{\|\cdot\|}$ and $\overline{B}^{\|\cdot\|}$ are subspaces of Y and Y^*, respectively, with $\Phi(B) \subset A$ and $\Psi(A) \subset B$. Since we have

- $\|y\| = \sup\{\langle y^*, y\rangle : y^* \in B \cap B_{Y^*}\}$ for every $y \in A$,
- $\{y^* \in \overline{B}^{w^*} : \langle y^*, y\rangle = 0 \text{ for every } y \in A\} = \{0\}$,

an appeal to [7, Lemma 6.1.1] ensures the existence of a bounded linear projection $P : Y \longrightarrow Y$ such that $P(Y) = \overline{A}^{\|\cdot\|}$ and $\langle y^*, y\rangle = 0$ for every $y^* \in B$ and every

$y \in \ker P$. It is clear that the subspaces $Y_0 := P(Y)$ and $Y_1 := \ker P$ satisfy the required properties. The last assertion of the lemma now follows easily. □

A measurable space is a pair (Ω, Σ), where Ω is a set and Σ is a σ-algebra on Ω. Given a countably additive measure ν defined on Σ with values in a Banach space Y, we write $|\nu|$ to denote the *variation* of ν, i.e. the function $|\nu| : \Sigma \longrightarrow [0, \infty]$ defined by $|\nu|(A) = \sup \sum_{i=1}^{n} \|\nu(A_i)\|$, where the supremum is taken over all the finite partitions $(A_i)_{i=1}^{n}$ of A in Σ. As usual, we say that $|\nu|$ is *σ-finite* if there is a countable partition (E_n) of Ω in Σ such that $|\nu|(E_n) < \infty$ for every $n \in \mathbb{N}$.

Let (Ω, Σ, μ) be a complete probability space and Y a Banach space. Recall that a function $f : \Omega \longrightarrow Y^*$ is said to be *Gelfand integrable* if for every $y \in Y$ the function $\langle f, y\rangle : \Omega \longrightarrow \mathbb{R}$, defined by $\omega \mapsto \langle f(\omega), y\rangle$, is μ-integrable; in this case, there exists (see e.g. [5, p. 53]) a finitely additive measure $\gamma_f : \Sigma \longrightarrow Y^*$ such that

$$\langle \gamma_f(A), y\rangle = \int_A \langle f, y\rangle \, d\mu \quad \text{for every } A \in \Sigma \text{ and every } y \in Y.$$

Notice that if f is bounded then γ_f is countably additive and has finite variation, because for each $A \in \Sigma$ we have

$$\|\gamma_f(A)\| = \sup_{y \in B_Y} \langle \gamma_f(A), y\rangle = \sup_{y \in B_Y} \int_A \langle f, y\rangle \, d\mu \le \Big(\sup_{\omega \in \Omega} \|f(\omega)\|\Big) \cdot \mu(A).$$

2. The results

In order to deal with Theorem 2.3 we need two auxiliary lemmas.

Lemma 2.1. *Let Y be a Banach space such that (B_{Y^*}, w^*) is separable. Let (Ω, Σ) be a measurable space and $\nu : \Sigma \longrightarrow Y$ a countably additive measure. Then $\nu(\Sigma)$ is norm separable.*

Proof. We know that $\nu(\Sigma)$ is relatively weakly compact (cf. [5, Corollary 7, p. 14]). Thus, in order to prove the lemma we only have to check that *any weakly compact set $K \subset Y$ is norm separable.* To this end, notice that the subspace $Z = \overline{\text{span}}(K) \subset Y$ is weakly compactly generated and, in particular, every bounded w^*-separable subset of Z^* is w^*-metrizable (Lemma 1.1). Since (B_{Y^*}, w^*) is separable, its continuous image (B_{Z^*}, w^*) is separable too. It follows that Z is norm separable, as required. □

Notice that any Banach space as in the previous lemma is isomorphic to a subspace of ℓ^∞. Therefore, Lemma 2.1 can also be obtained as a consequence of Rosenthal's theorem saying that, for any probability measure μ, every weakly compact subset of $L^\infty(\mu)$ is separable (cf. [5, Theorem 13, p. 252]).

Lemma 2.2. *Let X be a Banach space, $\mathcal{G}$ the family of all open sets of (B_{X^*}, w^*) and μ a Radon probability measure on (B_{X^*}, w^*). Then the 'identity' function $I : B_{X^*} \longrightarrow X^*$, $I(x^*) = x^*$, is Gelfand integrable with respect to μ and the support of μ is contained in $\overline{\text{span}}^{w^*}(\gamma_I(\mathcal{G}))$.*

Proof. The first assertion is obvious. Now fix $x_0^* \in B_{X^*} \setminus \overline{\text{span}}^{w^*}(\gamma_I(\mathcal{G}))$. By the Hahn-Banach separation theorem, there is $x \in X$ such that $\langle x^*, x\rangle = 0$ for every $x^* \in \gamma_I(\mathcal{G})$ and $\langle x_0^*, x\rangle > 0$. Take $\varepsilon > 0$ and $G \in \mathcal{G}$ containing x_0^* such that $\langle x^*, x\rangle \ge \varepsilon$ for every $x^* \in G$. Hence

$$0 = \langle \gamma_I(G), x\rangle = \int_G \langle x^*, x\rangle \, d\mu(x^*) \ge \varepsilon \mu(G)$$

and therefore $\mu(G) = 0$. It follows that x_0^* does not belong to the support of μ. □

We can now prove the main result of this paper.

Theorem 2.3. *Let X be a WLD Banach space. The following conditions are equivalent:*

(i) (B_{X^*}, w^*) *has property (M).*

(ii) *For every measurable space (Ω, Σ) and every countably additive measure $\nu : \Sigma \longrightarrow X^*$ with σ-finite variation, $\nu(\Sigma)$ is norm separable.*

In this case, for such a ν there exist a separable complemented subspace $X_0 \subset X$ and a complemented subspace $Z_0 \subset X^*$ isomorphic to X_0^* such that $\nu(\Sigma) \subset Z_0$.

Proof. (ii)⇒(i) Fix a Radon probability measure μ on (B_{X^*}, w^*) and let $S \subset B_{X^*}$ be its support. Since S is w^*-compact, in order to prove that S is w^*-separable we only have to check that S is w^*-metrizable. Consider the 'identity' mapping $I : B_{X^*} \longrightarrow X^*$ as a bounded Gelfand integrable function with respect to μ. Since γ_I is countably additive and has finite variation, it has norm separable range. On the other hand, by Lemma 2.2 we have $S \subset \overline{\text{span}}^{w^*}(\gamma_I(\mathcal{G}))$, where $\mathcal{G}$ is the family of all open sets of (B_{X^*}, w^*). Since $\gamma_I(\mathcal{G})$ is norm separable, $\overline{\text{span}}^{w^*}(\gamma_I(\mathcal{G}))$ is w^*-separable and Lemma 1.1 now ensures that S is w^*-metrizable. The proof of (ii)⇒(i) is over.

(i)⇒(ii) We divide the proof into several steps.

Step 1.- Since ν is countably additive, there is a non-negative countably additive measure μ on Σ such that $\lim_{\mu(A)\to 0} \|\nu(A)\| = 0$ (Bartle, Dunford and Schwartz [4], cf. [5, Corollary 6, p. 14]). Observe that we can assume without loss of generality that μ is complete. Indeed, if we write $(\Omega, \Sigma_0, \mu_0)$ to denote the completion of the finite measure space (Ω, Σ, μ), a standard argument would allow us to extend ν to a countably additive measure $\nu_0 : \Sigma_0 \longrightarrow X^*$ with σ-finite variation such that $\lim_{\mu_0(A)\to 0} \|\nu_0(A)\| = 0$.

Step 2.- Suppose that there is a constant $C > 0$ such that $|\nu|(A) \leq C\mu(A)$ for every $A \in \Sigma$. Then (see e.g. [6, Proposition 6.7]) there is a Gelfand integrable function $f : \Omega \longrightarrow X^*$ such that

- $f(\Omega) \subset CB_{X^*}$;
- $\gamma_f = \nu$;
- f is Σ-Borel(CB_{X^*}, w^*)-measurable;
- the completion of the image measure μf^{-1} on Borel(CB_{X^*}, w^*) is a Radon probability measure.

The fact that (CB_{X^*}, w^*) has property (M) allows us to find a w^*-separable set $T \in$ Borel(CB_{X^*}, w^*) such that $\mu(\Omega \setminus f^{-1}(T)) = 0$. An appeal to the Hahn-Banach separation theorem now establishes that $\nu(\Sigma) \subset \overline{\text{span}}^{w^*}(T)$ (bear in mind that $\nu = \gamma_f$). Notice that $\overline{\text{span}}^{w^*}(T)$ is w^*-separable.

Step 3.- Since ν has σ-finite variation, we can find a countable partition (A_n) of Ω in Σ such that, for each $n \in \mathbb{N}$, there is $C_n > 0$ such that $|\nu|(A) \leq C_n\mu(A)$ for every $A \in \Sigma$, $A \subset A_n$ (see e.g. the proof of Lemma 5.9 in [6]). In view of *Step 2*, for each $n \in \mathbb{N}$ the set $\{\nu(A) : \ A \in \Sigma, \ A \subset A_n\}$ is contained in a w^*-separable subset of X^*. Therefore, the same holds for the set

$$R = \Big\{ \sum_{n=1}^{N} \nu(E_n) : \ E_n \in \Sigma, \ E_n \subset A_n \text{ for every } 1 \leq n \leq N, \ N \in \mathbb{N} \Big\}.$$

Since ν is countably additive, R is norm dense in $\nu(\Sigma)$ and we conclude that $\nu(\Sigma)$ is contained in a w^*-separable set $S \subset X^*$.

Step 4.- According to Lemma 1.1, there exist two subspaces $X_0, X_1 \subset X$ such that $X = X_0 \oplus X_1$, X_0 is separable and $\langle x^*, x \rangle = 0$ for every $x^* \in S$ and every $x \in X_1$. Let us consider the bounded operators $\xi_i : X^* \longrightarrow X_i^*$ defined by $\xi_i(x^*) = x^*|_{X_i}$ for $i = 0, 1$. Set $Z_0 = \ker \xi_1$ and $Z_1 = \ker \xi_0$. It is easy to check that

$X^* = Z_0 \oplus Z_1$ and that the restriction $\xi_0|_{Z_0} : Z_0 \longrightarrow X_0^*$ is an isomorphism of Banach spaces. Since $(B_{X_0^{**}}, w^*)$ is separable (by Goldstein's theorem) and $\nu(\Sigma) \subset Z_0$, an appeal to Lemma 2.1 ensures that $\nu(\Sigma)$ is norm separable. The proof is complete. □

Theorem 2.3 can be translated easily into the language of operators:

Corollary 2.4. *Let X be a WLD Banach space. The following conditions are equivalent:*

(i) (B_{X^*}, w^*) *has property (M).*
(ii) *For every complete probability space (Ω, Σ, μ) and every bounded operator $T : L^1(\mu) \longrightarrow X^*$, $T(L^1(\mu))$ is norm separable.*

In this case, for such a T there exist a separable complemented subspace $X_0 \subset X$ and a complemented subspace $Z_0 \subset X^*$ isomorphic to X_0^* such that $T(L^1(\mu)) \subset Z_0$.

Proof. (i)⇒(ii) The formula $\nu(A) := T(\chi_A)$ (where χ_A stands for the characteristic function of $A \in \Sigma$) defines a countably additive measure $\nu : \Sigma \longrightarrow X^*$ such that $|\nu|(A) \leq \|T\|\mu(A)$ for every $A \in \Sigma$. The norm separability of $T(L^1(\mu))$ and the last assertion of the corollary now follow from Theorem 2.3 and the fact that simple functions are dense in $L^1(\mu)$.

(ii)⇒(i) Fix a Radon probability measure μ on (B_{X^*}, w^*) and consider again the 'identity' mapping $I : B_{X^*} \longrightarrow X^*$ as a bounded Gelfand integrable function with respect to μ. Clearly, there is a bounded operator $T : L^1(\mu) \longrightarrow X^*$ such that $T(\chi_A) = \gamma_I(A)$ for every $A \in \Sigma$. By hypothesis, T has norm separable range and so the same holds for γ_I. The proof finishes as in the implication (ii)⇒(i) in Theorem 2.3. □

Since the indefinite integral of any Pettis integrable function has σ-finite variation (see e.g. [12, Theorem 4.1]), our Theorem 2.3 can be applied to deduce:

Corollary 2.5. *Let X be a WLD Banach space such that (B_{X^*}, w^*) has property (M) (for instance, a weakly $\mathcal{K}$-countably determined space). Then X^* has the Pettis Separability Property.*

Combining Theorem 2.3 with the result of Rybakov mentioned in the introduction, we arrive at the following corollary.

Corollary 2.6. *Let X be a WLD Banach space. If X does not contain subspaces isomorphic to ℓ^1, then (B_{X^*}, w^*) has property (M).*

We finish the paper by showing that the absence of copies of $\ell^1(\omega_1)$ in a Banach space X is a necessary condition to have the property that every countably additive X^*-valued measure with σ-finite variation has norm separable range.

Example 2.7. *There exists a countably additive $\ell^\infty(\omega_1)$-valued measure with finite variation whose range is not norm separable.*

Proof. For each ordinal $\alpha < \omega_1$ we write e_α to denote the element of $\ell^1(\omega_1)$ given by $e_\alpha(\beta) = \delta_{\alpha,\beta}$ (the Kronecker symbol) and $\pi_\alpha : \{0,1\}^{\omega_1} \longrightarrow \mathbb{R}$ stands for the α-coordinate projection. Let us denote by $(\{0,1\}^{\omega_1}, \Sigma, \mu)$ the complete probability space obtained after completing the usual product probability measure on $\{0,1\}^{\omega_1}$. Consider the 'identity' function $f : \{0,1\}^{\omega_1} \longrightarrow \ell^\infty(\omega_1)$ and notice that $\langle f, e_\alpha \rangle = \pi_\alpha$ for every $\alpha < \omega_1$. Bearing in mind that f is bounded, we conclude that f is Gelfand integrable and that $\nu := \gamma_f$ is a countably additive measure with finite variation.

Set $E_\alpha = \pi_\alpha^{-1}(\{1\}) \in \Sigma$ for every $\alpha < \omega_1$. Then we have

$$\|\nu(E_\alpha) - \nu(E_\beta)\|_\infty \geq |\langle \nu(E_\alpha), e_\alpha\rangle - \langle \nu(E_\beta), e_\alpha\rangle|$$
$$= \Big| \int_{E_\alpha} \pi_\alpha \, d\mu - \int_{E_\beta} \pi_\alpha \, d\mu \Big| = \mu(E_\alpha) - \mu(E_\alpha \cap E_\beta) = \frac{1}{4}$$

whenever $\alpha, \beta < \omega_1$, $\alpha \neq \beta$. It follows that $\nu(\Sigma)$ is not separable. □

Given a subspace Y of a Banach space X and a countably additive Y^*-valued measure ν' with finite variation, a theorem of Musial and Ryll-Nardzewski [14] ensures that there is a countably additive X^*-valued measure ν with finite variation such that $r \circ \nu = \nu'$, where $r : X^* \longrightarrow Y^*$ denotes the 'restriction' operator. Thus, in view of Example 2.7, we obtain the announced result.

Corollary 2.8. *Let X be a Banach space with a subspace isomorphic to $\ell^1(\omega_1)$. Then there exists a countably additive X^*-valued measure with finite variation whose range is not norm separable.*

The converse of Corollary 2.8 does not hold in general. Indeed, under the Continuum Hypothesis, Kalenda (see [15, Corollary 4.4]) showed that there is a WLD Banach space X such that (B_{X^*}, w^*) does not have property (M). Thus we can find a countably additive X^*-valued measure with finite variation whose range is not norm separable (Theorem 2.3). On the other hand, $\ell^1(\omega_1)$ cannot be isomorphic to a subspace of X, because the property of being WLD is inherited by subspaces (see e.g. the remarks after Proposition 1.2 in [1]) and $\ell^1(\omega_1)$ is not WLD.

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