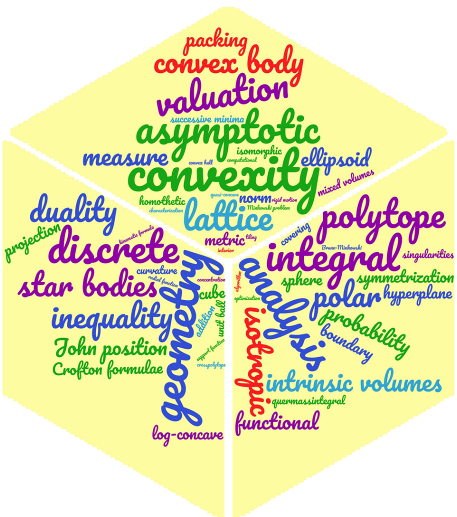
CONFERENCE

CURRENT TRENDS IN CONVEX GEOMETRY

March 15th-19th, 2021



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SCHEDULE

Monday March 15th

10:00-10:50 Monika Ludwig: *The Hadwiger theorem for convex functions*

11:00-11:20 Dmitry Faifman: *A Funk extension of the Blaschke-Santaló inequality*

11:30-11:50 Gil Solanes: *Santaló point for the Holmes-Thompson boundary area*

15:30-16:20 Elisabeth Werner: *Blaschke-Santaló inequality for many functions and geodesic barycenters of measures*

16:30-16:50 Iskander Aliev: *On the volume of central hyperplane sections of a d-dimensional cube*

17:00-17:20 Ansgar Freyer: *Slicing- and successive minima type-inequalities for the lattice point enumerator*

Tuesday March 16th

10:00-10:50 Shiri Artstein-Avidan: *On non-traditional measure transport*

11:00-11:20 Luis Carlos García: *Decompositions of polytopes and Banach-Stone-like theorems for metric spaces*

11:30-11:50 Georg Hofstätter: *Asplund Endomorphisms and the Blaschke–Santaló inequality*

15:30-16:20 Artem Zvavitch: *The convexification effect of Minkowski sum*

16:30-16:50 Arnaud Marsiglietti: *Geometric and functional inequalities for log-concave probability sequences*

17:00-17:20 Galyna Livshyts: *The equality case characterization in the Brascamp-Lieb inequality*

Wednesday March 17th

15:30-16:20 Alina Stancu: *The fundamental gap and convex sets in hyperbolic space revisited*

16:30-16:50 Yair Shenfeld: *Current progress on the Alexandrov-Fenchel inequalities*

17:00-17:20 Julián Pozuelo: *The Brunn-Minkowski inequality in Nilpotent Lie groups*

17:30-18:30 Poster session

Thursday March 18th

10:00-10:50 Andrea Colesanti: *About Brunn-Minkowski type inequalities for intrinsic volumes with respect to the p-addition,* $p < 1$

11:00-11:20 Nico Lombardi: *Plane R-paths and descent curves*

11:30-11:50 Liran Rotem: *Linear functionals on log-concave functions*

15:30-16:20 Dmitry Ryabogin: *On bodies floating in equilibrium in every direction*

16:30-16:50 Nicola Turchi: *High-dimensional beta polytopes: volume phase transition*

17:00-17:20 Luis Montejano: *On the isometric conjecture of Banach*

Friday March 19th

10:00-10:20 Ferenc Fodor: *The volume of central diagonal sections of the* n*-cube*

10:30-10:50 Silouanos Brazitikos: *Vector-Valued Maclaurin Inequalities*

11:00-11:50 Vitali Milman: *Trends in Convexity: non-linear convexity and flowers*

ABSTRACTS OF INVITED LECTURES

On non-traditional measure transport

Shiri Artstein-Avidan
(joint work with Shay Sadovsky and Kasia Wyczesany)

We discuss an analogue of the Brenier/McCann theorem on optimal measure transport for costs which are "non-traditional", in that they are allowed to assume infinite values. Our motivating example is the polar cost which induces the polarity transform. We provide a new Rockafellar-Rüschendorf theorem which holds for such costs, and apply it to prove that under weak assumptions on the cost, whenever two measures are strongly c-compatible, there exists an optimal transport plan which lies on a c-subgradient of a function in the appropriate class (all definitions and notions will be explained throughout the talk, as well as examples and intuition).

Tel Aviv University, Israel

About Brunn-Minkowski type inequalities for intrinsic volumes with respect to the p-addition, $p < 1$

Andrea Colesanti
(joint work with C. Bianchini, D. Pagnini and A. Roncoroni)

In this talk we consider the following question: do intrinsic volumes verify a Brunn-Minkowski type inequality with respect to the p-addition, for $p < 1$? We provide some partial (positive and negative) results.

Università degli Studi di Firenze, Italy

The Hadwiger theorem for convex functions

Monika Ludwig
(joint work with Andrea Colesanti and Fabian Mussnig)

A functional Z defined on a space of real-valued functions $\mathcal{F}$ is called a *valuation* if

$$\mathrm{Z}(f \vee g) + \mathrm{Z}(f \wedge g) = \mathrm{Z}(f) + \mathrm{Z}(g)$$

for all $f, g \in \mathcal{F}$ such that $f, g, f \vee g, f \wedge g \in \mathcal{F}$. Here $f \vee g$ is the pointwise maximum of f and g, while $f \wedge g$ is their pointwise minimum. The important, classical notion of valuations on convex, compact sets is a special case of the rather recent notion of valuations on function spaces.

We present a complete classification of all continuous, epi-translation and rotation invariant valuations on the space of super-coercive convex functions on $\mathbb{R}^n$. This result corresponds to Hadwiger's celebrated theorem on the classification of continuous, translation and rotation invariant valuations on the space of convex bodies. The valuations obtained in our theorem are functional versions of the classical intrinsic volumes.

Technische Universität Wien, Austria

Trends in Convexity: non-linear convexity and flowers

Vitali Milman

In the talk we describe the following subjects:

- Quadratic type equations (Il. Molchanov)
- Geometric mean; Power (V. M. – L. Rotem)
- Flower, reciprocity, flower-Minkowski summation, flower power (V. M., Emmanuel Milman, L. Rotem)
- Connection with Böröczky-Lutwak-Yang-Zhang (BLYZ)
- More non-linear constructions (V. M. - L. Rotem)

Tel Aviv University, Israel

On bodies floating in equilibrium in every direction

Dmitry Ryabogin

In this talk we will discuss known and recent results related to Ulam's Problem 19 from the Scottish Book, asking *is a solid of uniform density which will float in water in every position a sphere?*

Kent State University, USA

The fundamental gap and convex sets in hyperbolic space revisited

Alina Stancu

About a year ago I have reported on work with collaborators (T. Bourni, J. Clutterbuck, H. Nguyen, G. Wei and V. Wheeler) that resulted in showing that the classical fundamental gap formula fails to hold for geodesically convex domains in hyperbolic space. At the time, several questions ensued, in particular whether h-convexity should replace geodesic convexity to obtain a lower bound on the gap. H-convexity is considered the strongest among several notions of convexity in the hyperbolic space, which become equivalent in Euclidean space. In this talk, I will present the results on the fundamental gap of convex domains in hyperbolic space after considering different definitions of convexity, as well as other aspects of the problem that are different from the classical Euclidean space, as well as constant positive curvature case.

Concordia University, Canada

Blaschke-Santaló inequality for many functions and geodesic barycenters of measures

Elisabeth Werner
(joint work with A. Kolesnikov)

Motivated by the geodesic barycenter problem from optimal transportation theory, we prove a natural generalization of the Blaschke-Santaló inequality and the affine isoperimetric inequalities for many sets and many functions. We derive from it an entropy bound for the total Kantorovich cost appearing in the barycenter problem. We also show a monotonicity property of the multimarginal Blaschke-Santaló functional.

Case Western Reserve University, USA

The convexification effect of Minkowski sum

Artem Zvavitch
(joint work with Mokshay Madiman, Matthieu Fradelizi, Arnaud Marsiglietti and Zsolt Langi)

For a compact set A in $\mathbb{R}^n$ let $A(k)$ be the Minkowski sum of k copies of A scaled by $1/k$. It is intuitively clear and well known that $A(k)$ approaches the convex hull of A in Hausdorff distance as k goes to infinity.

In this talk we will discuss how exactly $A(k)$ approaches the convex hull of A, and more generally, how a Minkowski sum of possibly different compact sets approaches convexity, as measured by various indices of non-convexity. The non-convexity indices considered will include the Hausdorff distance induced by most general norm, the volume deficit (the difference of volumes), a non-convexity index introduced by Schneider, and the effective standard deviation or inner radius. We will present relationships between those indices and move to discussion of monotonicity of convergence of $A(K)$ with respect to those indices. In particular, we will present a conjecture proposed, a few years ago, by Bobkov, Madiman and Wang, that the volume of $A(k)$ is non-decreasing in k, or in other words, that when the volume deficit between the convex hull of A and $A(k)$ goes to 0, it actually does so monotonically. While this conjecture holds true in dimension 1, it turns out it fails in the dimension 12 or greater. We will also present a special case of this conjecture for star-shaped sets, and we will prove that the conjecture holds true as long as k is greater or equal than $n - 1$.

Kent State University, USA

ABSTRACTS OF SHORT LECTURES

On the volume of central hyperplane sections of a d-dimensional cube

Iskander Aliev

We present an optimal upper bound for the (normalised in a certain way) volume of a central hyperplane section of an origin-symmetric d-dimensional cube. This confirms a conjecture recently posed by Imre Barany and Peter Frankl. The proof is based on an application of Busemann's theorem.

Cardiff University, UK

Vector-Valued Maclaurin Inequalities

Silouanos Brazitikos

We investigate a MacLaurin inequality for vectors and its connection to an Aleksandrov-type inequality.

University of Athens, Greece

A Funk extension of the Blaschke-Santaló inequality

Dmitry Faifman

The Funk metric in the interior of a convex body is an example of a projective metric, and an affine-invariant relative of the more well-known projectively invariant Hilbert metric. A natural question one can ask is, how large can a ball of a given radius possibly be? We will attempt to give an answer, with the Holmes-Thompson definition of volume, and see how it interpolates between the Blaschke-Santaló and centro-affine surface area isoperimetric inequalities, and extends an inequality of K. Ball.

Tel Aviv University, Israel

The volume of central diagonal sections of the n-cube

Ferenc Fodor
(joint work with Ferenc A. Bartha and Bernardo Gonzales Merino)

We prove that the volume of central hyperplane sections of a unit cube in $\mathbb{R}^n$ orthogonal to a diameter of the cube is a strictly monotonically increasing function of the dimension for $n \geq 3$. The proof is based on an integral formula for central sections of the cube, and on Laplace's method which we use to estimate the asymptotic behaviour of the integral. We treat small dimensions by numerical methods.

University of Szeged, Hungary

Slicing- and successive minima type-inequalities for the lattice point enumerator

Ansgar Freyer
(joint work with Martin Henk)

We study inequalities on the number of integer points inside a convex body in terms of lower-dimensional structures, such as hyperplane-sections and -projections, as well as Minkowski's successive minima. We give a partial answer to a conjecture of Gardner, Gronchi and Zong on a discrete analog of Meyer's inequality. Also, a discrete version of Minkowski's second theorem, which is equivalent to the continuous original, is presented.

Technische Universität Berlin, Germany

Decompositions of polytopes and Banach-Stone-like theorems for metric spaces

Luis Carlos García Lirola
(joint work with M. Alexander, M. Fradelizi and A. Zvavitch)

Given two finite metric spaces, we characterize when there exists an isometry between the corresponding spaces of Lipschitz functions. To this end, the 1-Lipschitz functions are identified with a certain polytope in $\mathbb{R}^n$, whose polar is the so-called Kantorovich-Rubinstein polytope and which is very related to the graph structure of the metric space. We also study the possibility of decomposing these polytopes as an ℓ_1 or an ℓ_∞-sum and characterize when they are Hanner polytopes.

Universidad de Zaragoza, Spain

Asplund Endomorphisms and the Blaschke–Santaló inequality

Georg Hofstätter
(joint work with F. E. Schuster)

Minkowski endomorphisms, that is, continuous and Minkowski additive maps on convex bodies compatible with rigid motions, have long been a focus of interest in convex geometry. In this talk, we introduce the new notion of Asplund endomorphisms that generalizes Minkowski endomorphisms to the setting of (coercive) log-concave functions, extending Minkowski additivity to additivity with respect to the Asplund sum. We construct a large family of monotone Asplund endomorphisms, each restricting to a monotone Minkowski endomorphism on indicators of convex bodies.

Moreover, we prove a family of analytic inequalities for the constructed Asplund endomorphisms, each inequality being stronger than the functional Urysohn inequality. The strongest one among our new family of inequalities is the functional Blaschke–Santaló inequality (for even functions). By restricting the inequalities to indicators, corresponding geometric inequalities for monotone Minkowski endomorphisms (including the classical Urysohn inequality) are recovered in an asymptotically optimal form.

Technische Universität Wien, Austria

The equality case characterization in the Brascamp-Lieb inequality

Galyna Livshyts

We discuss the equality case characterization in the Brascamp-Lieb inequality. It utilizes a quantitative stability version, which is based on the L2 approach together with some tools from trace theory, and a compactness approximation argument. If time permits, we mention some strengthenings of the Brascamp-Lieb inequality, and the (curious) equality case characterizations for them.

Georgia Institute of Technology, USA

Plane R-paths and descent curves

NICO LOMBARDI
(joint work with Marco Longinetti, Paolo Manselli and Adriana Venturi)

Let $g\colon [a,b] \to \mathbb{R}^2$ be a continuous mapping, oriented according to the increasing variable. Let us assume that for every $x \in g([a,b])$ there is a constraint Q_x on g; then what extra properties does the mapping g satisfy?

Different constraints for the previous points have been considered by several authors and for each constraint different classes of family of curves have been studied. In this talk the following constraint will be considered: let $R > 0$; for every $t \in (a,b]$,

$$g([a,t]) \cap B_R(g(t) + Rl) = \varnothing,$$

where l is a unit vector in a suitable tangent set to g at $g(t)$ and $B_R(g(t) + Rl)$ is the open disk of radius R and centered in $x(t) + Rl$. These paths are called R-paths.

This family of paths satisfy several properties, as rectifiability and angle estimate, and in particular it will be considered the relation with the notion of descent curves of a nested family of sets with reach greater or equal to R.

Technische Universität Wien, Austria

Geometric and functional inequalities for log-concave probability sequences

ARNAUD MARSIGLIETTI

We investigate various geometric and functional inequalities for the class of log-concave probability sequences, such as dilation and concentration inequalities. Our proof techniques are of independent interest, we find that log-affine sequences are the extreme points of the set of log-concave sequences belonging to a half-space slice of the simplex. This amounts to a discrete analog of the localization lemma of Lovasz and Simonovits.

University of Florida, USA

On the isometric conjecture of Banach

Luis Montejano

The following is known as the geometric hypothesis of Banach: let V be an m-dimensional Banach space (over the reals or the complex numbers) with unit ball B and suppose all n-dimensional subspaces of V are isometric (all the n-sections of B are affinely equivalent). In 1932, Banach conjectured that under this hypothesis V is a Hilbert space (the boundary of B is an ellipsoid).

Gromow proved in 1967 that the conjecture is true for n even, and Dvoretzky derived the same conclusion under the hypothesis $n = \infty$ and V is a real Banach space. Moreover, V. Milman proved the corresponding case when V is a complex Banach space.

We prove this conjecture for $n = 4k + 1$, with the possible exception, in the real case, when $n = 133$. The ingredients of the proof are classical homotopy theory, irreducible representations of the orthogonal group and convex geometry.

Suppose B is an $(n+1)$-dimensional convex body with the property that all its n-sections through the origin are affinely equivalent to a fixed n-dimensional body K. Using the characteristic map of the tangent vector bundle to the n-sphere, it is possible to prove that if n is even, then K must be a ball, and using homotopical properties of the irreducible subgroups of $SO(n)$, $SU(n)$, we prove that K must be a body of revolution (with the possible exception of $n = 133$ in the real case). Finally, we prove, using convex geometry and topology that, if this is the case, then there must be a section of B which is an ellipsoid and consequently B must be also an ellipsoid.

Using the same techniques, we obtain analogues results for projections instead of sections.

Universidad Nacional Autónoma de México, México

The Brunn-Minkowski inequality in Nilpotent Lie groups

Julián Pozuelo

There has been a growing interest in the Brunn-Minkowski inequality and its relation with the isoperimetric inequality in the context of nilpotent Lie groups, where the Lebesgue measure is replaced by the Haar measure and the Minkowski addition of sets is defined using the group product.

In this talk, we shall give a direct proof of the inequality

$$|A * B|^{1/n} \geq |A|^{1/n} + |B|^{1/n},$$

where A, B and $A * B$ are measurable sets in $\mathbb{R}^n$, $|\cdot|$ is the Lebesgue measure and $A * B$ is the Minkowski addition of sets defined through a product of the form

$$z * w = z + w + (F_1, F_2(z,w), \dots, F_d(z,w)) = z + w + F(z,w),$$

where F_1 is a constant and F_i are continuous functions that depend only on $z_1, \dots, z_{i-1}$, $w_1, \dots, w_{i-1}$ $\forall i = 2, \dots, d$. As a consequence, we shall obtain the Brunn-Minkowski inequality in Nilpotent Lie groups.

Universidad de Granada, Spain

Linear functionals on log-concave functions

Liran Rotem

Let F is a functional mapping convex bodies to real numbers. If F is increasing and linear (with respect to the Hausdorff sum) then it is well known that $F(K)$ must be the integral of the support function of K with respect to a fixed measure on the sphere.

In this talk we will present a new functional extension of this theorem where "convex bodies" are replaced with "log-concave functions". We will see that an increasing, linear functional on the class of log-concave functions corresponds to *two* measures, one on the sphere and one on all of $\mathbb{R}^n$. As the motivating family of examples we will discuss the surface area measure of log-concave functions and see how this new theorem corresponds to previously known results by Colesanti-Fragalà and by myself.

Technion, Israel

Current progress on the Alexandrov-Fenchel inequalities

Yair Shenfeld

While the Alexandrov-Fenchel inequalities form the backbone of convex geometry, the characterization of their equality cases, which are solutions to generalized isoperimetric problems, has been open for decades. In this talk I will briefly review the content of the recent preprint (joint work with Ramon van Handel): "The extremals of the Alexandrov-Fenchel inequality for convex polytopes" where we settle many of these open problems as well as problems in combinatorics. Time permitting, connections to broader Hodge theories will be discussed.

MIT, USA

Santaló point for the Holmes-Thompson boundary area

Gil Solanes
(joint work with Florent Balacheff and Kroum Tzanev)

For a convex body K, the volume of the polar body $(K-x)^\circ$ with respect to x is minimized when x is the classical Santaló point of K. This happens precisely when $(K-x)^\circ$ has its centroid at the origin. Similarly, one can look for points minimizing the Euclidean area of $\partial(K-x)^\circ$, or more generally its Holmes-Thompson area with respect to some norm. In the talk we will present a recent joint work with Florent Balacheff and Kroum Tzanev where this direction is explored. In particular we show that there exists a unique minimizer when the norm is sufficiently smooth. For Minkowski norms, we also obtain a characterization in terms of a certain dual centroid.

Universitat Autònoma de Barcelona, Spain

High-dimensional beta polytopes: volume phase transition

Nicola Turchi

A beta polytope is the convex hull of N i.i.d. random points in the d-dimensional Euclidean unit ball according to the so-called beta probability distribution, which has recently attracted considerable attention in stochastic geometry. The uniform distribution in the ball and the uniform distribution on the sphere are particular cases of such a distribution. The normalised volume (w.r.t. the volume of the unit ball) of the beta polytope is a random variable taking values in $[0,1]$. We study the sequence consisting of the expectations of these random variables as the space dimension diverges. Intuitively if N grows very slowly with d then such expectation will tend to 0, on the other hand it will go to 1 if N grows extremely fast. We show what is the number of points $N(d)$ that marks the threshold for the change of the limit behaviour and what is the shape of the phase transition. Results concerning intrinsic volumes and number of vertices are also shown.

Université du Luxembourg, Luxembourg

ABSTRACTS OF POSTERS

Optimal divisions of a convex body

ANTONIO CAÑETE
(joint work with Isabel Fernández and Alberto Márquez)

For a convex body C in $\mathbb{R}^n$ and a given division of C into $C_1, \ldots, C_n$ convex subsets, we can consider $\max\{F(C_1), \ldots, F(C_n)\}$ (respectively, $\min\{F(C_1), \ldots, F(C_n)\}$), where F represents one of these classical geometric functionals: the diameter, the width, the inradius and the circumradius. In some sense, the previous value provides a measure of the quality of the division. In this work we will study the divisions of C minimizing (respectively, maximizing) the previous value. In particular, we will treat the existence, uniqueness and balancing behaviour of the optimal divisions, bounds for the corresponding optimal values, and algorithms leading to these optimal divisions.

Universidad de Sevilla, Spain

On the optimal constants in the two-sided Stechkin inequalities

THOMAS JAHN

We address the optimal constants in the strong and the weak Stechkin inequalities, both in their discrete and continuous variants. These inequalities appear in the characterization of approximation spaces which arise from sparse approximation or have applications to interpolation theory. Improvements of the best known constants and proofs of the minimal constants are given through direct computation or through numerical methods for convex optimization problems.

TU Chemnitz, Germany

New Brunn-Minkowski and isoperimetric inequalities for the lattice point enumerator

Eduardo Lucas Marín
(joint work with David Iglesias and Jesús Yepes Nicolás)

The classical Brunn-Minkowski inequality in the n-dimensional Euclidean space asserts that the volume (Lebesgue measure) to the power $1/n$ is a concave functional when dealing with convex bodies (non-empty compact convex sets). It quickly yields, among other results, the classical isoperimetric inequality, which can be summarized by saying that the Euclidean balls minimize the surface area measure (Minkowski content) among those convex bodies with prescribed positive volume.

There exist various facets of the previous results, due to their different versions, generalizations and extensions. In this poster we will discuss and show discrete analogues of the above inequalities for the lattice point enumerator, which provides with the number of integer points of a given set, when dealing with arbitrary bounded subsets of the n-dimensional Euclidean space. Moreover, we will show that these new discrete inequalities imply the corresponding classical results for non-empty compact sets.

Universidad de Murcia, Spain

On Grünbaum type inequalities

Francisco Marín Sola
(joint work with Jesús Yepes Nicolás)

Given a compact set $K \subset \mathbb{R}^n$ of positive volume, and fixing a hyperplane H passing through its centroid, we find a sharp lower bound for the ratio $\mathrm{vol}(K^-)/\mathrm{vol}(K)$, depending on the concavity nature of the function that gives the volumes of cross-sections (parallel to H) of K, where K^- denotes the intersection of K with a halfspace bounded by H. When K is convex this inequality recovers a classical and powerful result by Grünbaum, whose generalizations and extensions to other settings are of high interest nowadays in Geometric Analysis and beyond. In this poster we will recall the well-known Grünbaum inequality and we will show our results.

Universidad de Murcia, Spain

Characterization of centrally symmetric convex bodies in terms of visual cones

Efren Morales Amaya

In this work we prove the following result. Let K be a strictly convex body in the Euclidean space E^n, $n > 2$, and let L be a hypersurface which is an embedding of the sphere S^{n-1} such that K is contained in the interior of L. Suppose that for every cone C_x, with apex at the point x in L, which circumscribes K, there exists a point y in L and a vector p in E^n such that the equality $C_y = p + C_x$ holds, where C_y is the cone with apex at the point y in L and circumscribes K. Then K and K are centrally symmetric and concentric.

Autonomous University of Guerrero, Mexico

Metrics and isometries for convex functions

Fabian Mussnig
(joint work with Ben Li)

We introduce functional analogs of the symmetric difference metrics for convex functions and give a full classification of all isometries.

Università degli Studi di Firenze, Italy

On the monotonicity of the isoperimetric quotient for parallel bodies

Christian Richter
(joint work with Eugenia Saorín Gómez)

The isoperimetric quotient of the whole family of inner and outer parallel bodies of a convex body is shown to be decreasing in the parameter of definition of parallel bodies, along with a characterization of those convex bodies for which that quotient happens to be constant on some interval within its domain. This is obtained relative to arbitrary gauge bodies, having the classical Euclidean setting as a particular case. Similar results are established for different families of Wulff shapes that are closely related to parallel bodies. These give rise to solutions of isoperimetric-type problems. Furthermore, new results on the monotonicity of quotients of other quermassintegrals different from surface area and volume, for the family of parallel bodies, are obtained.

Friedrich Schiller Universität Jena, Germany

Isodiametric problem in the spherical and hyperbolic spaces

Adam Sagmeister

My poster is about the isodiametric problem in the spherical and hyperbolic spaces where the optimal solutions are balls similarly to the Euclidean case. Even some stability versions are provided.

ELTE Budapest, Hungary