



Functions of Several Real Variables II

Syllabus

Course code:	1579
Number of ECTS credits:	6
Semester:	1st (September-January)
Prerequisites:	None
Recommended components:	Functions of one real variable I (1568), Functions of one real variable II (1573), Topology of Metric Spaces (1575), Linear Algebra (1569) Affine and Euclidean Geometry (1574)
Language of instruction:	Spanish (students are allowed to ask questions and write homeworks and exams in English)

Course description

This is one of the three courses corresponding to the subject Mathematical Analysis of Several Variables. All the topics are related to the integration of functions of several real variables. In particular, we cover Riemann and Lebesgue integration in \mathbb{R}^n , the basics of Measure Theory and applications of integration. Although we emphasize theoretical aspects, we also care the explicit computation of integrals, or getting good estimates otherwise.

Learning outcomes and competences

After completion of this course you will:

1. be able to compute integrals of several variables, or estimate its value sharply.
2. know the basics of measure theory and Lebesgue integration.
3. know several useful transformations of coordinates and its use in Analysis.
4. get some acquaintance with the applications of the integral, outside of Mathematics.

Course contents

I. Riemann and Riemann-Stieltjes Integral

1. Riemann integration of functions of several variables.
Negligible sets, Lebesgue characterization of Riemann integrability
2. Theorem of Fubini
Iterated integration in simple domains. Elementary change of variables. Applications

3. Introduction to Riemann-Stieltjes integration.

Integration by parts formula

II. Measure of sets

1. Abstract measure theory

σ -algebras, measures, outer measures.

2. Lebesgue measure.

Regularity and geometrical properties. Existence of non-measurable sets.

III. Lebesgue Integral

1. Measurable functions

Properties, approximation by simple functions

2. Integrable functions

Convergence theorems, completeness of spaces of integrable functions.

3. Improper and parametric integrals

Functions defined by integrals, applications.

IV. Product measure

1. Lebesgue measure as a product measure.

Iterated integration. Applications

V. Change of variables

1. Image measure.

Ramdon variables and expectation

2. Theorem of change of variables

Linear transformations, differentiable transformations, Sard's theorem, applications

References

Main texts

1. E. Stein, R. Shakarchi, *Real Analysis: Measure Theory, Integration, and Hilbert Spaces*, Princeton University Press, Princeton, 2005
2. Cohn, D. L. *Measure Theory*, Birkhäuser, 2013.

Supplementary references

1. R.L. Wheeden, A. Zigmund. *Measure and Integral*, Marcel Dekker, New York, 1977
2. A. Pallarés, *Teoría de la Medida*, <http://webs.um.es/apall> y Aula Virtual de la UM, 1999
3. M. de Guzmán, B. Rubio. *Integración : teoría y técnicas*. Alhambra, Madrid, 1979
4. G. Vera, *Lecciones de Análisis Matemático II*, <http://ocw.um.es/ciencias/analisis-matematico-ii>, 2005
5. J.A. Fernández Viña. *Análisis Matemático III*, Tecnos. Madrid 1992
6. W. Rudin. *Análisis Real y Complejo*, McGraw Hill, 1987.