UNIVERSIDAD DE MURCIA

FACULTY OF MATHEMATICS

Linear programming

Syllabus

Course code:	1582
Number of ECTS credits:	6
Semester:	1st (September-January)
Prerequisites:	None
Recommended components:	Linear Algebra (1569)
Language of instruction:	Spanish (students are allowed to ask questions and write homeworks and exams in English)

Course description

This is an introductory course on formulating mathematical models and developing solution methods for real-life optimal decision problems. We will study how to obtain the best decisions (according to a well-defined objective) in allocating scarce resources such as capital, materials, equipment, manpower, energy, etc. among competing activities that produce goods and services. Rather than developing a specific solution method for each optimization problem, we will build abstractions of these problems in the form of mathematical models and study a general method to solve these models.

The course will focus on a class of problems that can be modeled as a Linear Programming Model. Formally, a linear programming model is either a minimization or maximization of a linear function of several variables constrained with linear inequalities. Surprisingly, a large number of decision problems fit into this framework. This explains why linear programming is so widely used in a variety of industries, ranging from transportation to health care, from finance to manufacturing. The methodological development will include the simplex algorithm, theorems of duality, complementary slackness and sensitivity analysis.

Learning outcomes and competences

After completion of this course you will:

1. be able to recognize situations that LP can be used and formulate problems as LP's.
2. know the details about simplex algorithm and its various implementation issues and be able to implement them in solving LP.
3. know duality theory, why it is important, and be able to use it to derive results related to LP.
4. know how to address sensitivity analysis questions along with their underlying explanations.
5. be able to solve LP problems using Mosel/Xpress solver.

Course contents

I. Fundamentals of Linear Programming

1. Linear programming models.
   General setting. Canonical formulation. Standard formulation. Graphical method.
2. Convexity topics.
   Convex sets. Definition and properties. Extreme points and extreme directions. Convex combinations. Convex hull. Separating hiperplanes. Alternative theorems.
3. Polytopes.
   Basic solution. Extreme points: characterization and existence. Extreme directions. Characterization. Theorem of representation. Extreme directions existence. Fundamental theorem of Linear Programming.

II. Algorithms

1. Simplex algorithm.
   Extreme point optimality test. Improving a solution. Unicity and multiplicity of solutions. Algorithm scheme. Stalling and cycling. Simplex tableau.
2. Obtaining initial solutions.
   Artificial variables. 2-phases method. Penalty method.
3. Simple algorithm variants.
   Revised algorithm. Revised simplex tableau. Transforming problems with bounded variables to the standard form. Extreme point characterization and optimality test. Algorithm for problems with bounded variables.

III. Duality and sensitivity analysis

1. Dualility in Linear Programming.
   Dual problem formulation. Weak duality theorem. Strong duality theorem. Slackness complementarity theorem. Solving the dual problem through the primal problem. Economical interpretation.
2. Dual simplex algorithm.
   Dual feasible base. Changing the base. Scheme algorithm. Obtaining the initial base.
3. Sensitivity and parametric analysis.
   Changes in the data and optimality. Cost vector sensitivity. Resources vector sensitivity. Changes in the constraints. Vector cost parametric analysis. Resources vector parametric analysis.

References

Main texts

1. Bazaraa M.S., Jarvis J.J. y Sherali H.D. *Linear programming and network flows (3rd edition)*; Limusa, 1998.

Supplementary references

1. Williams H.P. *Model building in Mathematical Programming (3rd edition)*; Wiley, 1998.
2. Chang Y.-L. *WinQSB - Decision support software for MS/OM*; Wiley & Sons, 1998.
3. Guéret C., Prins C. y Sevaux M. *Applications of optimization with Xpress-MP*; Dash Optimization, 2002.
4. Ríos S., Ríos D., Mateos A. y Martín J. *Programación lineal y aplicaciones. Ejercicios resueltos*; Rama, 1997.
5. Luenberger D.G. *Programación Lineal y No Lineal*; Addison-Wesley, 1989.
6. Mocholi Arece M. y Sala Garrido R. *Programación lineal. Metodología y problemas*; Tebar Flores, 1993
7. Pardo L. *Programación Lineal Continua - Aplicaciones prácticas en la empresa*; Ediciones Díaz de Santos, 1987.
8. Hillier F.S. y Lieberman G.J. *Introducción a la Investigación de Operaciones*; McGraw-Hill, 1997.