Groups and Rings

Syllabus

Course code:	1585
Number of ECTS credits:	6
Semester:	2nd (Febryary-June)
Prerequisites:	None
Recommended components:	Linear Algebra (1569), Sets and Numbers (1570), Affine and Euclidean Geometry (1574)
Language of instruction:	Spanish (students are allowed to ask questions and write homeworks and exams in English)

Course description

This course is an introduction to abstract algebra and more concretely to two of the most relevant algebraic structures: groups and rings.

The goal of the course is to strengthen the competence of comprehension and usage of abstract concepts, as well as to develop the ability of analysis and rigour in the understanding of proofs and problem-solving. More concretely, the aim is to acquire ability in the manipulation of the most basic algebraic objects: groups and rings.

Learning outcomes and competences

After completion of this course you will:

1. be able to recognize relevant elements and subsets in groups and rings;

2. manage elements and substructures in groups and rings;

3. be able to construct quotient groups and quotient rings, and to work with their elements;

4. know and manage elements and structures in outstanding families of groups and rings;

5. know how to relate groups and rings through homomorphisms;

6. be able to apply the isomorphism theorems for groups and rings;

7. be able to decide, in easy cases, whether two groups or two rings are isomorphic, by the use of properties that are invariant thruogh isomorphisms;

8. know the basic decomposition of abelian groups;

9. be able to apply the classification of finite abelian groups to describe all abelian groups of a given size;

10. recognize the prime and maximal ideals in notable families of domains;

11. know how to construct the field of fractions of a domain, and appreciate its importance;

12. manage properties of divisibility and factorization in commutative domains;

13. know and identify in easy examples the euclidean domains, the principal ideal domains and the unique factorization domains;

14. be able to solve problems, adjusted to the level of the course, about the abstract theory of groups and rings;

15. be able, in the field of study, to think out conjectures and to imagine strategies to prove or disprove them.

Course contents

I. Groups

Groups, subgroups and homomorphisms; fisrt examples. Lagrange theorem. Order of an element; cyclic groups.

Symmetric groups; cycles and factorization of permutations. Parity, alternating groups. Dihedral groups.

Conjugation and class equation. Normal subgroups and quotient groups; isomorphism theorems; Abel theorem.

II. Finite abelian groups

Direct sums. Finite abelian groups: indecomposable decompositions; p-groups; indecomposable groups. Structure theorems: primary and invariant decompositions; presentations by generators and relations.

Introduction to finitely generated abelian groups and their structure theorems.

III. Rings

Rings. Ring homomorphisms. Subrings. Ideals, operations with ideals. Quotient rings. Isomorphism theorems. Maximal and prime ideals. Rings of fractions, localization.

IV. Factorization in domains

Divisibility in domains. Irreducible and prime elements. Principal ideal domains. Unique factorization domeins. Euclidean domains.

V. Polynomial rings

Polynomial rings. Polynomial over a field. Factorization of polynomials, polynomials over unique factorization domains, irreducibility criteria, cases $\mathbb{Q}$, $\mathbb{R}$ y $\mathbb{C}$. Zeroes of polynomials.

References

Main texts

1. Dorronsoro, J.; Hernandez, E. *Numeros, grupos y anillos.* Addison-Wesley/Universidad Autonoma de Madrid. 1996.

2. Bhattacharya, P.B.; Jain, S.K.; Nagpaul, S.R. *Basic Abstract Algebra.* Cambridge University Press, 1986.

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Supplementary references

1. del Valle A., del Rio, A., Simon, J. J. *Algebra Basica.* Universidad de Murcia - Diego Marin.

2. Clark, A. *Elementos de Algebra Abstracta.* Alhambra, 1974.

3. Hungerford, T.W. *Algebra.* Springer, 1974.

4. Anzola, M.; Caruncho, J.R. *Problemas de algebra, tomo 1: Conjuntos, estructuras.* Los autores D.L. 1981.

5. Anzola, M.; Caruncho, J.R., Perez-Canales, G. *Problemas de algebra, tomo 2: Anillos, polinomios, ecuaciones.* Los autores D.L. 1981.

6. Delgado F., Fuertes C., Xambo, S. *Introduccion al Algebra *.* Ed. U. Complutense 1993.

7. Delgado F., Fuertes C., Xambo, S. *Introduccion al Algebra ** Anillos, factorizacion y teoria de cuerpos.* Ed. U. Valladolid, 1998.

8. Delgado F., Fuertes C., Xambo, S. *Introduccion al Algebra *** Soluciones de los problemas.* Ed. U. Valladolid, 2000.