



# Topology of surfaces

## Syllabus

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<b>Course code:</b>	1587
<b>Number of ECTS credits:</b>	6
<b>Semester:</b>	2nd (February-June)
<b>Prerequisites:</b>	None
<b>Recommended components:</b>	1570 and 1575
<b>Language of instruction:</b>	Spanish (students are allowed to ask questions and write homeworks and exams in English)

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### Course description

In this course we first introduce the concept of abstract topological space and study some of its more important properties like connectedness and compactness. After that, we concentrate our study on topological surfaces with the aim of establishing the fundamental classification theorem for compact surfaces, as well as studying the concepts of homotopy and fundamental group. This course is closely related to the course of topology of metric spaces, already done in the first year, and with the course of geometry of curves and surfaces.

### Learning outcomes and competences

After completion of this course you will:

1. Know the concept of topological space and the most important examples.
2. Know the concepts of connectedness and compactness and how to determine if a given topological space satisfies them.
3. Know the concept of topological surface and understand the classification theorem of compact surfaces,
4. Know the concept of homotopy and its relationship with the topological classification of spaces.
5. Know the concept of fundamental group and its relationship with homotopy.

### Course contents

#### I. Topological spaces

1. Topological spaces: Topological spaces. More examples of topological spaces.
2. Topological properties: Connectedness. Compactness. Separation Axioms.

3. Homeomorphisms and topological constructions: Homeomorphisms. Product spaces. Quotient spaces.

## II. Homotopy and fundamental group

1. Homotopy: Homotopy. Homotopy equivalence. The circle. Brouwer's fixed-point theorem. Vector fields.
2. The fundamental group: Fundamental group. Induced homomorphisms. Van Kampen theorem.

## III. Surfaces

1. The Euler number: Simplicial complexes. Euler number. The Euler characteristic and surfaces.

## References

1. Kinsey, L. Christine. Topology of surfaces. Undergraduate Texts in Mathematics. Springer-Verlag, New York, 1993.
2. Adams, Colin ; Franzosa, Robert. Introduction To Topology: Pure and Applied. Prentice Hall.
3. Munkres, James R. Topology: a first course. Prentice-Hall, Inc., Englewood Cliffs, N.J., 2001.
4. Crossley, Martin D. Essential topology. Springer Undergraduate Mathematics Series. Springer-Verlag London, Ltd., London, 2005. 1-85233-782-6.
5. Armstrong, Mark Anthony. Basic topology. Undergraduate Texts in Mathematics. Springer-Verlag, New York-Berlin, 1983.