UNIVERSIDAD DE **MURCIA**

FACULTY OF **MATHEMATICS**

Geometry of Curves and Surfaces

Syllabus

Course code:	1589
Number of ECTS credits:	6
Semester:	1st (September-January)
Prerequisites:	None
Recommended components:	You should be familiar with Linear Algebra, Calculus of several variables and Topology.
Language of instruction:	Spanish (students are allowed to ask questions and write homeworks and exams in English)

Course description

This course is devoted to study Differential Geometry of curves and surfaces. The history of Differential Geometry starts with curves: notions like tangent of a curve date back to the times of Euclides, Arquímedes or Apolonio. Euclides himself introduced, in Definition 5 of his famous *Elements*, the concept of surface: "a surface is what has only length and breadth", stating in that way the idea of a surface as a 2-dimensional object, this is, something that can be described using two variables. However, one could say that Euler (1760) was the founder of the Theory of Surfaces in the 3-dimensional space. This course is the first contact of the student with Differential Geometry. Therefore, a good understanding of the contents of this topic will be important in order to study the future courses in Differential Geometry, both the obligatory and the optional ones.

Learning outcomes and competences

After completion of this course you will:

1. be able to arc-length parameterize a curve and to compute its curvature and torsion.
2. know how solve problems on curves using the Frenet formulae.
3. know the fundamental theorems of curves.
4. know and easily manage the main concepts in Theory of Surfaces (regular surface, differential of a map, tangent plane, first and second fundamental forms).
5. distinguish the different type of curvatures of a surface.
6. know the different type of points of a surface.
7. know special kind of curves that one can find in a surface: lines of curvature and asymptotic curves.
8. know and be able to apply the main result in Theory of Surfaces: the Egregium theorem of Gauss.

Course contents

I. Curves in the plane and the space.

1. Preliminaries. *Parameterized curves. Arc-length. Regular curves. Arc-length parameterization.*
2. Local theory of planar curves. *Frenet frame. Signed curvature. Frenet formulae. Fundamental theorem of planar curves. Comparing a curve and its tangent line in a point. Comparing two tangent curves in a point.*
3. Local theory of curves in the space. *Frenet frame. Curvature and Torsion. Frenet formulae. Fundamental theorem of curves.*

II. Surfaces in the space.

1. Preliminaries. *Regular surface. Level surface. Change of parameters.*
2. Differentiability. *Differentiable functions on surfaces. Differentiable maps between surfaces. Diffeomorphisms.*
3. The tangent plane. *Differentiable curves on a surface. Tangent vectors. The tangent plane of a surface. First fundamental form.*
4. The differential of a map. *Differential of a map. Gradient. Critical points. Differential of a differentiable map between surfaces. Chain rule. Inverse function theorem.*

III. Curvature.

1. Orientation of surfaces. *Vector fields on surfaces. Orientable surfaces. Complex structure. Positively and negatively oriented bases.*
2. The second fundamental form. *The Gauss map. Spherical image. The Weingarten endomorphism. Second fundamental form.*
3. Normal curvatures. *Acceleration of a curve on a surface. Extrinsic and intrinsic accelerations. Normal curvature. Normal sections. Relation between the normal curvature and the curvature of the normal section.*
4. Principal curvatures. *Principal curvatures and associated principal directions. Euler formula. Lines of curvature.*
5. Gauss' curvature and mean curvature. *The Gauss curvature and the mean curvature as algebraic invariants of the Weingarten endomorphism. Umbilical points. The Gauss and the mean curvatures in local coordinates. Totally umbilical surfaces and their characterization.*
6. Geodesic curvature. *Darboux trihedron. Geodesic curvature. Relation between the geodesic, the normal and the usual curvatures of a curve. Geodesic torsion. Darboux formulae. Asymptotic curves.*

IV. The Egregium theorem of Gauss.

1. Isometries. *Length of a curve on a surface and first fundamental form. Intrinsic and extrinsic geometry. Local and global isometries.*
2. Gauss and Weingarten formulae. *Christoffel symbols. Gauss' formulae. Weingarten's formulae.*
3. Compatibility equations and Gauss' Egregium theorem. *Mainardi-Codazzi equations. Gauss' equation. Gauss' Egregium theorem. Isometries and Gauss' curvature.*
4. The fundamental theorem of surfaces. *Bonnet's theorem.*

References

Main texts

1. Hernández Cifre, M. A. and Pastor González, J. A. *Un curso de Geometría Diferencial (A course of Differential Geometry)* (Spanish); Publicaciones del CSIC, Textos Universitarios 47, Madrid, 2010.

2. do Carmo, M. P. *Differential Geometry of Curves and Surfaces*; Prentice-Hall, Inc., Englewood Cliffs, N.J., 1976.

Supplementary references

1. Montiel, S. and Ros, A. *Curves and surfaces*; 2nd edition, translated from the 1998 Spanish edition by the authors. Graduate Studies in Mathematics, 69. American Mathematical Society, Providence, RI; Real Sociedad Matemática Española, Madrid, 2009.

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