UNIVERSIDAD DE **MURCIA**

FACULTY OF **MATHEMATICS**

Graphs and Discrete Optimization

Syllabus

Course code:	1592
Number of ECTS credits:	6
Semester:	1st (September-January)
Prerequisites:	None
Recommended components:	Linear Optimization (1582)
Language of instruction:	Spanish (students are allowed to ask questions and write homeworks and exams in English)

Course description

This is an introductory course on (i) undirected graph theory and (ii) formulating mathematical models and developing solution methods for real-life optimal decision problems when all or some of the decision variables are discrete.

In the first part we will study the main characteristics of an undirected graph, including the main theoretical results and solution algorithms. In the second part, we will study how to obtain the best solutions (according to a well-defined objective) or simply a feasible solution in problems defined on graphs (shortest paths, minimum spanning trees, minimum number of colors. . .) or not (knapsack problems, discrete location problems, set packing problems, sudoku and many more). An important class of these problems is the one resulting of using binary variables, which are frequently used to modelize decisions yes/not which are present in most of the real-life and theoretically-oriented optimization problems.

Learning outcomes and competences

After completion of this course you will:

1. know the main undirected graphs and their properties;
2. be able to count and/or enumerate many classes of important graphs;
3. develop theoretical properties of graphs;
4. be able to recognize situations in which Discrete Optimization can be used, and formulate the resulting problems;
5. know the details about branch-and-bound algorithms and cutting plane methods;
6. be able to solve the problems using a commercial solver.

Course contents

I. Graph Theory

1. Introduction to Graph Theory.
   Notation, definitions, graph representation, first theoretical results.
2. Connectivity.
   Connected graphs, properties and algorithms, k-connected and k-edge-connected graphs, relation between connectivities, separation, Menger's theorem.
3. Trees.
   Characterization of trees, properties, binary trees, minimum spanning tree and algorithms.
4. Paths and tours.
   Networks. Shortes paths in a network: theoretical properties and algorithms; Eulerian tours, the Chinese Postman Problem, Hamiltonian cycles, Dirac's theorem, Chvátal theorem, the Traveling Salesman Problem.
5. Coloring and planarity.
   Graph coloring, chromatic number, chromatic polyhedron, reduction theorem, planar graphs, embedding, Euler's formula, maximal planar graphs, Kuratowski's theorem, 4/5 colors theorems.

II. Discrete Optimization

1. The model.
   Discrete/Integer Optimization/Programming Model, intensive formulation, total unimodularity, solution with commercial solvers.
2. Branch and bound. Cutting hyperplanes.
   Branch-and-bound algorighm for Discrete Optimization, all-integer cutting planes, mixed cutting planes, Chvátal-Gomory inequalities, sufficienty of CG inequalities.

References

Main texts

1. Diestel R. *Graph Theory*; Electronic Edition, 2000.
2. Wolsey L. *Integer Programming*; Wiley, 1998.

Supplementary references

1. Alsina C. *Mapas del metro y redes neuronales. La teoría de grafos*; RBA, 2011.
2. Townsend. *Discrete Mathematics: Applied combinatorics and Graph Theory*; Benjamin/Cummings, 1987.
3. Salkin, Mathur. *Foundations of integer programming*; North-Holland, 1989.