UNIVERSIDAD DE **MURCIA**

FACULTY OF **MATHEMATICS**

Partial Differential Equations and Fourier series

Syllabus

Course code:	1593
Number of ECTS credits:	6
Semester:	2nd (January-June)
Prerequisites:	None
Recommended components:	Ordinary Differential Equations (1584)
Language of instruction:	Spanish (students are allowed to ask questions and write homeworks and exams in English)

Course description

The main aim of this course is to introduce the student in the knowledge and analysis of partial differential equations, a subject initiated some centuries ago and which has inspired the solution of many problems from the applied sciences and has helped to solve others current theoretical problems.

As a first topic it is developed the theory and applications of partial differential equations of first order underlying its geometrical origins and the possibility of geometric solving. Then it is faced the problem of linear partial differential equations of second order and coefficients depending on two variables. One of the main tool to develop, is using changes of variable in order to simplify them and obtain the simplest possible expressions called normal forms of the equations. When it is possible, we face the Cauchy problems associated to them by the method of series developping which is adequate and convenient for future numerical analysis.

In any number of independent variables, we consider the problems of linear partial differential equations of second order and constant coefficients and relate them to the paradigmatic examples of models taking form Mathematical Physics. Such approach implies to classify them into three groups: eliptic, parabolic and hyperbolic and study the Laplace equations, heat transmission equations and waves equations. In such problems we concentrate in the associated initial-boundary conditions, using mainly the variables separation method.

To faced such problems we developed the Fourier series theory and consider several types of orthonormal bases such as trigonometric, Bessel, trigonometric-Bessel and orthogonal polynomials. The problem of convergence of Fourier series in general conditions is also addressed. Finally we introduce the Fourier transform in order to solve some of the former problems and also to give the student a more general view and adequate interpretations.

Learning outcomes and competences

After completion of this course you will:

1. be able to recognize situations that partial differential equations can be used and formulate problems with such a tool.
2. to use with familiarity the variables separation methods
3. to apply the developed theory to other topics, such that non-linear wave transmission, musical producers,etc
4. to understand the formulation of important physics theories, such as Relativity and Quantic Mechanics
5. be prepaired to address numerical analysis of initial-boundary problems in partial differential equations.

Course contents

I. Introduction to Partial Differential Equations

   1. First definitions.
      Partial differential equations and their solutions. Order of a equation. Linear and quasi-linear equations.
   2. On Manifols of solutions
      General solutions. Cauchy and initial-boundary problems. Change of variable. Examples.

II. Partial diferential equations of first order.

   1. First examples.
      Transport equation. Fundamental conservative laws.
   2. Quasi-linear equations of first order with two independent variables
      Statement of the problem. Geometric interpretation. Characteristic curves, data curves and transversality condition. Cauchy problem; existence and uniqueness. Methods of finding solutions.

III. Introduction to partial differential equations of second order

   1. Hyperbolic, eliptic and parabolic equations.
      Classification of linear equations of secon order. Reduction to normal forms of equations with two independent variables. Normal forms with more than two independent variables. Normal forms for equations with constant coefficients and n independent variables.

IV. Introduction to Fourier series

   1. Introduction to Trigonometric Fourier series
      Coefficients and series. The class $\mathfrak{D}$ of Dirichlet maps. Pointwise convergence. Absolute and uniform convergence of Fourier series and their derivatives. The Parseval identity. Sine and Cosine Fourier series.
   2. Introduction to Bessel maps and orthogonal polynomials
      Bessel differential equation. Bessel maps of first and second class. Fourier-Bessel series. Differential equations whose solutions are orthogonal polynomials. Fourier series. Construction of spheric armonics.

V. Introduction to Fourier and Laplace transforms.

  1. Properties, examples and applications to the resolution of partial differential equations.

VI. Hyperbolic equations.

  1. Vibrant string.
     Physical statement. Formal construction of solutions. Verification of the validity of solutions found with variables separation. Uniqueness and continuity of solutions with respect to initial-boundary data.

VII. Parabolic equations.

  1. The transmission heat problem in a rod
     Physical statement. Formal constructions of solutions. Verification of the validity of solutions found with variables separation. Uniqueness and continuity of solutions with respect to initial-boundary data.

VIII. Eliptic equations

  1. Stationary temperature in a rectangle plate
     Physical statement. Formal constructions of solutions. Verification of the validity of solutions found with variables separation. Uniqueness and continuity of solutions with respect to initial-boundary data. The equation in polar coordinates and solution. The maximum principle for Laplace equation.

References

Main texts

1. Broman A. *Introduction to Partial Differential Equations. From Fourier Series to Boundary-value problems*; Dover New York, 1989.

2. John F. *Partial Differential Equations. Fourth edition*; Springer New York, 1982.

3. Peral I.*Primer Curso de Ecuaciones en Derivadas Parciales*; Addison-Wesley/UAM, 1995.

Supplementary references

1. Colton D. *Partial Differential Equations. An Introduction*; Random House, New York,1988.

2. Folland G.B.*Partial Differential Equations*; Princeton University Press, Princeton, 1976.

3. Tolstov G.P. *Fourier Series*; Dover, New York, 1976.

4. Weinberger H.*Curso de Ecuaciones en Derivadas Parciales con MÃ©todos de Variable Compleja y Transformaciones Integrales*; RevertÃ©, Barcelona, 1970.