UNIVERSIDAD DE **MURCIA**

FACULTY OF **MATHEMATICS**

Global Geometry of Surfaces

Syllabus

Course code:	1594
Number of ECTS credits:	6
Semester:	2nd (February-June)
Prerequisites:	None
Recommended components:	Geometry of curves and surfaces (1589) You should be familiar with Linear Algebra, Calculus of several variables and Topology.
Language of instruction:	Spanish (students are allowed to ask questions and write homeworks and exams in English)

Course description

Whenever one thinks about global problems in Differential Geometry, the usual strategy is to impose some *global* condition on the surface in order to avoid that open subsets of larger surfaces can appear as possible solutions. These conditions can be topological (e.g., compactness) or geometric (for instance, geodesic completeness). This course is devoted to study several essential results in Global Differential Geometry, like the Hopf-Rinow and the Gauss-Bonnet theorems.

Learning outcomes and competences

After completion of this course you will:

1. be able to know and to characterize a very special kind of curves that one can find in a surface: the geodesics.
2. know how to integrate functions on surfaces and how to compute the area of a surface region.
3. understand the curves/surfaces variations.
4. deeply know the Hopf-Rinow theorem and some of its applications.
5. know and be able to apply Gauss-Bonnet's theorem.

Course contents

I. Geodesics in surfaces.

   1. The parallel transport. *Vector fields along curves. Covariant derivative. Parallel fields. Existence and uniqueness theorem of parallel fields. The parallel transport.*
   2. Geodesics. *Geodesics and first properties. Existence and uniqueness theorem of geodesics.*

3. The exponential map. *The exponential map. Normal coordinates and geodesic polar coordinates. Gauss' lemma. Geodesics as the shortest path. Minding's theorem.*

II. Variational calculus in surfaces.

1. Integration over surfaces. *Area element. Integrating functions over surfaces. A geometric interpretation of the Gauss curvature.*
2. Area variations. *Definition of area variation. Minimal surfaces as solutions of a variational problem.*
3. Arc length variations. *Variation of curves and variational field. First and second variation of arc length. Geodesics as solutions of a variational problem. Bonnet's theorem.*

III. Completeness. The Hopf-Rinow theorem.

1. Complete surfaces. *The intrinsic distance. Minimization properties of geodesics. Geodesic completeness and metric completeness.*
2. The Hopf-Rinow theorem. *Hopf-Rinow's theorem. Geodesically complete surfaces. Compactness and completeness. Completeness and length of divergent curves.*

IV. The Gauss-Bonnet theorem.

1. The local version of Gauss-Bonnet's theorem. *The external angle at a vertex. Theorem of turning tangents. Total geodesic curvature. Gauss-Bonnet's formula. Applications.*
2. The global version of Gauss-Bonnet's theorem. *The Euler characteristic. The Gauss-Bonnet theorem. Consequences.*

References

Main texts

1. Hernández Cifre, M. A. and Pastor González, J. A. *Un curso de Geometría Diferencial (A course of Differential Geometry)* (Spanish); Publicaciones del CSIC, Textos Universitarios 47, Madrid, 2010.

2. do Carmo, M. P. *Differential Geometry of Curves and Surfaces*; Prentice-Hall, Inc., Englewood Cliffs, N.J., 1976.

Supplementary references

1. Montiel, S. and Ros, A. *Curves and surfaces*; 2nd edition, translated from the 1998 Spanish edition by the authors. Graduate Studies in Mathematics, 69. American Mathematical Society, Providence, RI; Real Sociedad Matemática Española, Madrid, 2009.