UNIVERSIDAD DE **MURCIA**

FACULTY OF **MATHEMATICS**

AlgebraicEquations

Syllabus

Course code:	1596
Number of ECTS credits:	6
Semester:	2nd (February-June)
Prerequisites:	None
Recommended components:	You should be familiar with the subjects *Sets and Numbers* and *Groups and Rings*.
Language of instruction:	Spanish (students are allowed to ask questions and write homeworks and exams in English)

Course description

The subject's general objective is the study of polynomial equations in one variable, based on their relationship (more or less evident) with the extensions of fields and on their more subtle relationship with group theory (Galois theory).

To achieve this objective, we develop the necessary theory and algebraic techniques. As a result, we apply the theory and techniques to the analysis of specific and generic polynomial equations and to the resolution of two classic problems: the constructions with ruler and compass and the solvability of equations by radicals.

Learning outcomes and competences

After successful completion of this course you will:

1. Know how to operate with polynomials in one and several variables.
2. Know and apply irreducibility criteria in polynomial rings.
3. Know how to express symmetric polynomials in terms of elementary symmetric polynomials.
4. Know and properly use the relationships between the coefficients of a polynomial equation in one variable and its roots.
5. Manipulate expressions involving algebraic and transcendent elements on the rational field. Know how to calculate the irreducible polynomial of algebraic elements.
6. Know how to relate geometric constructions with algebraic extensions.
7. Know the basic properties of normal and separable field extensions and know how to calculate splitting fields of polynomials.
8. Know the structure of finite fields and know how to manipulate their extensions.

9. Know the notion of the Galois group of an extension and how to calculate it in simple cases.

10. Know the fundamental theorems of Galois theory and the Lagrange's accessory irrationalities theorem, and apply them to obtain information about the Galois group and the intermediate fields of simple extensions, in particular finite and Galois extensions.

11. Know how to calculate primitive elements of simple extensions.

12. Know the concepts of cyclotomic extension and cyclic extension.

13. Know the concept of solvability by radicals of an equation and relate it to the notions of solvable-by-radicals extensions and of solvable groups.

14. Know and apply the specific techniques for the calculation of Galois groups of equations of degrees 3 and 4, and to solve them by radicals.

Course contents

I. Polynomials; symmetric polynomials.

Polynomial rings (review): Universal property; polynomials over fields and over unique factorization domains; factorization, irreducibility and roots of polynomials in one variable over the rational, real and complex fields.

Resolution by radicals of cubic and quartic equations.

Symmetric polynomials and symmetric rational functions. Cardano-Vieta formulas.

II. Field extensions.

Field extensions, degree of an extension. Adjunction of roots, Kronecker's theorem. Algebraic and transcendent elements, algebraic extensions.

III. Ruler and compass constructions.

Ruler and compass constructions, constructible numbers. Wantzel's theorem; application to classical problems: trisecting angles, squaring circles and doubling cubes.

IV. Splitting fields; algebraic closure; finite fields.

Splitting fields of polynomials. Finite fields. Algebraically closed fields. Algebraic closure.

V. Normal extensions and separable extensions.

Normal extensions. Normal closure. Separable extensions and perfect fields.

VI. Galois theory. Fundamental theorems.

The Galois group of an extension, Galois connection. Galois extensions. Finite Galois extensions: first and second fundamental theorem of Galois theory, Lagrange's accessory irrationalities theorem and the primitive element theorem.

VII. Cyclotomic extensions and cyclic extensions.

Roots of unity. Cyclotomic extensions. Regular polygons which are cconstructible with ruler and compass. Cyclic extensions, Hilbert's theorem 90.

VIII. Solvability of radical equations.

Radical extensions. Solvable groups and solvability by radicals of polynomial equations, Galois theorem. The general equation of degree n, Abel's theorem. Galois group and radical resolution of cubic and quartic equations.

References

Main texts

1. Asensio, J.; Caruncho, J.R.; Martínez, J.: Ecuaciones Algebraicas. Universidad de Murcia - DM, 2002. ISBN 9788484252603.

Supplementary references

1. Bhattacharya, P.B.; Jain, S.K.; Nagpaul, S.R.: Basic Abstract Algebra. Cambridge University Press, 1986. ISBN 9780521460811.
2. Tradacete Pérez, P.: Problemas resueltos de estructuras algebraicas: grupos, anillos y cuerpos. Bubok, D.L. 2010. ISBN 9788499168791.
3. Hungerford, T.W.: Algebra. Springer-Verlag, 1989. ISBN 9781461261018.
4. Stewart, I.: Galois Theory, Fourth Ed. Chapman and Hall, 2015. ISBN 9781482245820. eISBN 9781482245837.
5. Clark, A.: Elementos de Algebra Abstracta. Alhambra, 1974.ISBN 9788420504803.

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