UNIVERSIDAD DE
MURCIA

FACULTY OF
MATHEMATICS

Commutative Algebra

Syllabus

Course code:	1600
Number of ECTS credits:	6
Semester:	1st (September-January)
Prerequisites:	None
Recommended components:	Linear Algebra (1569), Sets and numbers (1570)
Language of instruction:	Spanish

Course description

Commutative algebra is essentially the study of commutative rings. Interest in this study come from two historical sources, Algebraic Geometry and Number Theory. Both of them remains between its main motivations and applications although these are not your only fields of application. The goal of this course is to complete the basic knowledge of Algebra started with the previous courses "Groups and Rings" (1585) and "Algebraic Equations" (1596).

Learning outcomes and competences

After completion of this course you will:

1. Know the definition and basic properties of some special types of ideals over commutative rings. To be able to determine the character of an ideal in concrete examples.

2. Knowing the definition and basic properties of the modules, and the classification of finitely generated modules over principal ideal domains. To be able to classify finite abelian groups and endomorphisms of vector spaces of finite dimension.

3. Know the definitions, properties and basic constructions of localization and the integral dependence, and know manipulate concrete examples.

4. Know some applications of commutative rings to algebraic geometry and algebraic number theory.

Course contents

I. Rings and ideals.
Review of some elementary properties of rings, ideals and rings homomorphisms. Prime and maximal ideals. Prime radical and Jacobson radical. Operations with ideals.

II. Modules.
Modules and modules homomorphisms. Submodules and quotient modules. Sums and direct products. Free modules. Finitely generated modules. Nakayama lemma. Noetherian and Artinian rings and modules.

III. Localization.
Rings and modules of quotiens. Local properties. Prime Spectrum and localizations of rings.

IV. Modules over principal ideal domains.
Free modules over a DIP. Matrices and elementary operations. Decomposition theorems. Uniqueness of the decompositions. Applications to abelian groups of finite type and classification of endomorphisms of a finite dimensional vector space.

V. Integral Dependence. Dedekind domains.
Normal extensions. The going-up theorem. Normal domains. The going-down theorem. Discrete valuation rings. Dedekind domains. Applications: Noether's normalization lemma. Hilbert's Nullstellensatz. Rings of integer of numbers fields.

References

- ATIYAH-MACDONALD. Introduccin al Algebra Conmutativa. Reverte, 2005.
- BALCERZYK-JOSEFIAK. Commutative Noetherian and Krull Rings. Ellis Horwood Limited 1989.
- SHARP. Steps in commutative algebra (2nd ed.). Cambridge Univ. Press, 2000.
- MATSUMURA. Commutative ring theory. Cambridge Univ. Press, 1992.
- REID. Undergraduate commutative algebra. Cambridge Univ. Press, 1997.
- BHATTACHARYA-JAIN-NAGPAUL. Basic Abstract Algebra. Cambridge University Press 1986.