UNIVERSIDAD DE **MURCIA**

FACULTY OF **MATHEMATICS**

Riemannian Geometry

Syllabus

Course code:	1602
Number of ECTS credits:	6
Semester:	1st (September-January)
Prerequisites:	None
Recommended components:	Geometry of curves and surfaces (1589) and Global Geometry of Surfaces (1594).
Language of instruction:	Spanish (students are allowed to ask questions and write homeworks and exams in English)

Course description

Riemannian geometry historically appears as an attempt to generalize the differential geometry of curves and surfaces in Euclidean space whose intrinsic character is given by the first fundamental form. Georg Friedrich Bernhard Riemann undertook this task by associating to each point a quadratic form which plays the role of such a fundamental form for surfaces, which was the origin of what nowadays we call Riemann metrics. Thus, the subject considered is a generalization to arbitrary dimension of the concepts studied in the third year of the Bachelor of Mathematics, "Geometry of curves and surfaces" and "Global Geometry of Surfaces".

Learning outcomes and competences

After completion of this course you will:

1. manage basic concepts associated with the notion of differentiable manifold.
2. compute the differential of a differentiable map.
3. know the general concept of tensor and, in particular, vector fields and differential forms.
4. manage basic concepts associated with the notion of Riemannian manifold.
5. handle the most interesting examples, in particular regular surfaces.
6. compute the parallel transport of a vector.
7. recognize curves which are geodesics.
8. know the minimizing properties of geodesics and the Hopf-Rinow theorem.
9. know the Riemann curvature tensor and its main properties.
10. know the notion of submanifold and, in particular, that of hypersurface.
11. manage the second fundamental form, as well as the Gauss and Weingarten formulae.
12. identify totally geodesic, totally umbilical and minimal hypersurfaces.

Course contents

I. Differentiable manifolds

Charts. Differentiable manifold. Tangent vectors. Tangent space. Covectors and cotangent space. Differentiable maps. The differential map. Diffeomorphisms. Vector fields. Differentiable forms. Tensors.

II. Riemannian manifolds

Metric tensors. Riemannian manifold. The Levi-Civita connection. The covariant derivative. Parallel transport. Geodesics. The exponential map. Distance associated to a metric. Minimizing properties of geodesics. Completeness. The Hopf-Rinow theorem. The Riemann curvature tensor. Properties. Sectional, Ricci and scalar curvatures.

III. Riemannian submanifolds

Isometric immersions. Isometric embeddings. Submanifolds. Hypersurfaces. Examples. Tangent and normal vector fields. The induced connection. The second fundamental form. The Gauss equation. The Weingarten endomorphism. The Weingarten formula. Totally geodesic and totally umbilical hypersurfaces. Minimal hypersurfaces.

References

Main texts

1. M. P. do Carmo, *Riemannian Geometry*, Birkhäuser, 1992.

2. B. O'Neill, *Semi-Riemannian Geometry*, Academic Press, Inc. 1983.