UNIVERSIDAD DE **MURCIA**

FACULTY OF **MATHEMATICS**

Numerical and variational methods for partial differential equations

Syllabus

Course code:	1603
Number of ECTS credits:	6
Semester:	1st (September-January)
Prerequisites:	None
Recommended components:	You should be familiar with ordinary and partial differential equations from, know some of the numerical ideas and methods for ordinary differential equations, calculus of several variables and with computer programming.
Language of instruction:	Spanish (students are allowed to ask questions and write homeworks and exams in English)

Course description

This course is a natural continuation to the numerical methods studied for ordinary differential equations. Here we will study the modern approach to partial differential equations.

After a basic review of the main basic types of pdes and boundary conditions we will give a short introduction to the finite difference methods. Some computational labs are proposed using MATLAB.

Next, an introduction to the theory of distributions, Sobolev spaces and weak formulations for pdes will be made. As a natural consequence of this setting, Galerkin methods will appear and, as a aparticular case, we will have the finite element method as the course benchmark.

We will work the basic rudiments for elliptic and parabolic problems and implement several examples using finite element methods in the 2d and 3d case by using the software FreeFem++. Our final purpose is to check on some examples for the Navier-Stokes equations for fluids.

The practical side of this course is about the 40% out of the total and, as we mentioned before, computer labs will be developed using FreeFem++ and MATLAB.

Learning outcomes and competences

After completion of this course you will:

1. be able to recognize the basic differential operators and boundary conditions and their meaning in terms of physical phenomena.
2. know the basics of the modern theory of partial differential equations,
3. know the basics and some details on the use of the finite difference methods,
4. know to some extent the principle and applications of the finite element methods.

Course contents

I. Introduction to partial differential equations and the finite difference method

1. Conservation laws. Transport, diffusion and reaction equations. Boundary conditions. Initial value and boundary value problems
2. Finite difference method for the basic equations. Implementation with MATLAB.

II. Modern theory of pdes

1. Basic functional analysis. Distribution theory, Sobolev spaces, Lax-Milgram lemma.
2. Galerkin approach. Basic convergence theory
3. Variational formulation of elliptic problems.
4. Variational formulation of parabolic problems.

III. Finite element method

1. Introduction to FreeFem++
2. Eliptic and parabolic problems.
3. Some examples of finite element spaces.
4. Navier-Stokes equations.

References

Main texts

1. Raviart, P.A., Thomas, J.M., *Introduction a l'analyse numérique des équations aux dérivées partielles*; Editorial Dunod, 2005.

2. Allaire, G. *Numerical Analysis and Optimization*; Oxford University Press, 2007.

3. Hecht, F. *FreeFem++ manual*, www.freefem.org

4. Johnson, C. *Numerical solution of partial differential equations by the finite element method*; Cambrigde University Press, 1987.

5. P. Solin, *Partial Differential Equations and the Finite Element Method*; Wiley 2006

Supplementary references

1. Strikwerda, J.C. *Finite Difference Schemes and Partial Differential Equations*; 2nd Edition, SIAM 2004

2. Brenner-Scott, *The Mathematical Theory of Finite Element Methods*; Springer 2008

3. Larson-Bengzon, *The Finite Element Method: Theory, Implementation and Practice*; Springer 2010