Foundations of Mathematics

Syllabus

Course code:	1607
Number of ECTS credits:	6
Semester:	2nd (February-June)
Prerequisites:	None
Recommended components:	None
Language of instruction:	Spanish (students are allowed to ask questions and write homeworks and exams in English)

Course description

The foundations of mathematics include elementary arithmetic, set theory and mathematical logic. We shall study the properties of arithmetic and set theory as formalized theories, by using the methods of mathematical logic.

Learning outcomes and competences

After successful completion of this course you will:

1. know the basic concepts and methods of first-order logic.

2. know the most usual axiomatization (ZFC) of set theory and have some information about other axiomatizations.

3. know the basics about ordinal and cardinal numbers.

4. be aware of the incompleteness theorems and understand their significance.

Course contents

I. Propositional Calculus.

   Propositional formulas. Valuations. Tautologies. Deduction. Compacity and completeness of the propositional calculus.

II. First-order languages.

   Syntactic properties of first-order languages. Deduction. Completeness and compacity.

III. Theories and models.

   First-order theories and their models. Gödel's completeness theorem. Löwenheim-Skolem theorems. Ultrafilters and ultraproducts. Non-standard analysis.

IV. Computability.

   Computable functions. Recursion theory. Recursively enumerable and decidable sets. Hilbert's tenth problem.

 V. Incompleteness theorems.

   The incompleteness of arithmetic. The consistency of arithmetic.

VI. Introduction to set theory.

   ZFC axiomatic of set theory. Ordinals and cardinals. The von Neumann universe.

References

Main texts

1. Y. Manin *Mathematical Logic*; Springer.

2. E. Mendelson *Introduction to Mathematical Logic (3rd Ed.)*; Wadsworth & Brooks.

3. J. Roitman *An introduction to modern set theory*; Wiley & Sons.

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