UNIVERSIDAD DE MURCIA

FACULTY OF MATHEMATICS

Geometry and Relativity

Syllabus

Course code:	1609
Number of ECTS credits:	6
Semester:	2nd (February-June)
Prerequisites:	None
Recommended components:	Riemannian Geometry (1602)
Language of instruction:	Spanish (students are allowed to ask questions and write homeworks and exams in English)

Course description

This course is one of the best examples of integration between two scientific disciplines: mathematics and physics. Specifically, we will take what we have learnt from other courses in differential geometry to try to understand how our universe is, both locally and globally. Using the mathematical tools we will be able to unravel, for example, what happens when you fall into a black hole, what kind of path you will follow, how to travel to the future, how light bends when it passes close to a massive object, how the shape of the universe is seen as a whole, how old it is, will it expand forever? ...

Learning outcomes and competences

After completion of this course you will:

1. Understand the principles of special relativity and its implications on the basic concepts of space and time.
2. Manage the concept of spacetime, understanding how it splits into space and time relative to an observer.
3. Understand how matter and energy curve spacetime and describe the paths of particles and light rays in curved spacetime.
4. Study the universe globally in order to determine their topological and geometric properties: finite, infinite, open, closed, curvature, etc.
5. Know the relativistic model for a star and its neighborhood, in particular the model for a black hole.
6. Understand the singularity theorems of Hawking and Penrose.

Course contents

I. Lorentz geometry

Lorentz metrics. Causal character of vectors. Timecones. Time orientation. Examples. Timelike curves and null curves. Causality. Lorentzian distance. Causal geodesics.

II. Special relativity

Newtonian space and time. Minkowski spacetime. Particles and photons. Observers. Lorentz-Fitzgerald contraction. Time dilation and length contraction. Twin paradox. Energy-momentum.

III. General relativity

Spacetimes. Particles and photons. Observers. Electromagnetic fields. Stress-energy tensor. Maxwell equations. The Einstein equation. Schwarzschild geometry. Orbits. Perihelion advance. Bending of light. Stellar collapse. Black holes.

IV. Cosmology

Perfect fluids. Robertson-Walker spacetimes. Friedmann models. Redshift. Hubble's law. Causality. Globally hyperbolic spacetimes. Cauchy hypersurfaces. Hawking's singularity theorem. Trapped surfaces. Penrose's singularity theorem.

References

Main texts

1. O'Neill, B., *Semi-Riemannian Geometry*; Academic Press, 1983.
2. Beem, J.K., Ehrlich, P.E. and Easley, K.L., *Global Lorentzian Geometry*; Marcel Dekker, 1996.

Supplementary references

1. Hawking, S.W. and Ellis, G.F.R., *The large scale structure of space-time*; Cambridge University Press, 1973.
2. Sachs, R.K. and Wu, H., *General Relativity for Mathematicians*; Springer-Verlag, 1977.
3. Faber, R.L., *Differential Geometry and Relativity Theory*; Marcel Dekker, 1983.
4. Rindler, W., *Relativity: Special, General, and Cosmological*; Oxford University Press, 2006.