UNIVERSIDAD DE MURCIA

FACULTY OF MATHEMATICS

Sets and Numbers

Syllabus

Course code:	6072
Number of ECTS credits:	6
Semester:	1st (September-January)
Prerequisites:	None
Recommended components:	None
Language of instruction:	Spanish (students are allowed to ask questions and write homeworks and exams in English)

Course description

The objective of the course is twofold. It aims to introduce the elementary notions of Set Theory which give the common language for all branches of Mathematics and, at the same time, some basic topics on numbers and polynomials are also presented. On the other hand, the introduction of these contents gives the opportunity for trying to make the student familiar with the rigorous methods of reasoning and the accuracy in studying and presenting results in Mathematics.

Learning outcomes and competences

After successful completion of this course you will:

1. know the most elementary notions and terms about sets, maps and binary relations.
2. appreciate how the basic properties of the different sets of numbers (integers, rationals, reals and complexes) can be derived from a small list of assumed properties about the natural numbers.
3. be able to construct correct proofs, elaborate arguments and obtain accurate results in Mathematics.
4. know the first properties of basic algebraic structures (abelian groups, rings, domains, fields) and experience how these abstract notions may help in obtaining information on the properties of the studied sets of numbers.
5. have a first approximation to the properties of real or complex polynomials and understand the ties and analogies between these properties and those of the integer domain.

Course contents

I. Sets

Terminology and notations on sets. Intersection, union, complements. Cartesian products.

II. Binary relations.

Relations. Ordering relations. Partitions. Equivalence relations. Quotients sets.

III. Functions.

Correspondences and functions. Composition of functions. Injective, surjective and bijective functions. Inverse functions. Binary operations. Compatibility of operations and equivalences.

IV. Natural numbers.

Basic properties. Induction principle. Applications to combinatorics. The euclidean division. Prime numbers and the fundamental theorem of Arithmetic. Gcd and Lcm. Denumerable and non-denumerable sets.

V. Integers.

The construction of the ring of integers. Divisibility and factorization of integers. The euclidean algorithm. Residue classes and modular arithmetic. The chinese remainder theorem and Euler's theorem.

VI. Sets of numbers.

The construction of the field of rational numbers. Cauchy sequences and the construction of the real numbers. Properties of the field of real numbers. The construction of the complex numbers.

VII. Polynomials.

Construction and structure of the ring of polynomials. Divisibility, irreducible polynomials and factorization. Factorization of real and complex polynomials.

References

Main texts

1. Notes in the web page of the course. University of Murcia.
2. F. Zaldívar, Fundamentos de álgebra, FCE - Universidad Autónoma Metropolitana (México), 2005.

Supplementary references

1. R. S. Irving, Integers, polynomials and rings, Springer Verlag (New York), 2004.
2. M. Anzola, J. Caruncho, Problemas de álgebra, Tomo I, (Madrid), 1981.