UNIVERSIDAD DE MURCIA

FACULTY OF MATHEMATICS

# Algebraic Geometry

## Syllabus

| | |
|---|---|
| **Course code:** | 6363 |
| **Number of ECTS credits:** | 6 |
| **Semester:** | 2nd (February-June) |
| **Recommended components:** | It is recommended to have attended and passed courses similar in contents to the undergraduate courses on 'Linear Algebra' (1569) and 'Groups and Rings' (1585) taught in the Mathematics Degree at the University of Murcia. The preparation of the student would be optimal, but not strictly necessary, if she/he would have attended courses similar in contents to those of the courses 'Algebraic Equations' (1596) and 'Commutative Algebra' (1600) of the mentioned Degree. |
| **Language of instruction:** | Spanish (students are allowed to ask questions and write homeworks and exams in English) |

## Course description

Algebraic Geometry is the meeting point of Geometry and Algebra. Classically, it deals with the study of real or complex curves and surfaces defined by polynomial equations and, moving further from physical intuition, with the study of subsets of $\mathbb{R}^n$ or $\mathbb{C}^n$ defined by the vanishing of polynomial equations with real or complex coefficients. The basic idea of the discipline is to tackle geometric problems for these curves, surfaces or hypersurfaces by using algebraic tools.

The course will be based on this classical approach, but generalizing the context to the case when the ground field is an arbitrary field $k$ and not just $\mathbb{R}$ or $\mathbb{C}$. That is, we will deal from the beginning with polynomial equations with coefficient in such a (fixed) field $k$. In the final chapter of the course, we will outline an introduction to schemes, a concept introduced and developed by Grothendieck in the 1960s which is the most useful tool in modern times to deal with problems of Algebraic Geometry.

## Learning outcomes and competences

After completion of this course you will:

1. have learnt to manipulate algebraic varieties, with particular emphasis on the affine and projective ones.

2. be able to use an algebro-geometric dictionary which will enable you to interpret geometric properties in algebraic terms, and viceversa.

3. be able to understand, both in an intutitive and rigorous way, the concept of dimension of a variety.
4. have a valuable introduction to the theory of schemes.

## Course contents

I. SECTION 1: Algebraic varieties

1. Affine varieties.
2. Projective varieties
3. Dimension of varieties.

II. SECTION 2: Morphisms of varieties

1. Regular functions
2. Rational functions
3. How to classify algebraic varieties?

III. SECTION 3: Regularity

1. Regular and singular points of an algebraic variety
2. Nonsingular varieties
3. Resolution of singularities.

IV. SECTION 4: Algebraic schemes

1. Sheaves
2. Algebraic schemes
3. Algebraic varieties as algebraic schemes.

## References

### BIBLIOGRAPHY

1. HARTSHORNE, R.: *Algebraic Geometry.* Springer-Verlag, 1993.
2. KUNZ, E.: *Introduction to Commutative Algebra and Algebraic Geometry.* Birkhäuser, 1985.
3. FULTON, W.: *Curvas Algebraicas.* Reverté, 1971
4. SHAFAREVICH, I.R.: *Basic Algebraic Geometry*, volume I. Springer-Verlag, 1994.
5. PERRIN, D.: *Algebraic Geometry, an introduction.* Springer-Verlag, 2008.
6. HOLME, A.: *A royal road to Algebraic Geometry.* Springer, 2012.
7. GRIFFITHS, P.; HARRIS, J.: *Principles of Algebraic Geometry.* John Wiley and Sons, 1994.
8. GÖRTZ, U.; WEDHORN, T.: *Algebraic Geometry I: Schemes with examples and exercises.* Vieweg-Teubner, 2010.

## ONLINE REFERENCES


1. FULTON, W.: *Algebraic curves*, http://www.math.lsa.umich.edu/~wfulton/CurveBook.pdf
2. GRATHMANN, A.: *Algebraic Geometry*, http://www.mathematik.uni-kl.de/~gathmann/class/alggeom-2002/main.pdf
3. VAKIL, R.: *Foundations of Algebraic Geometry*, http://math.stanford.edu/~vakil/216blog/FOAGjun1113public.pdf
4. MUMFORD, D.; ODA, T.: *Algebraic Geometry II*, http://www.dam.brown.edu/people/mumford/alg_geom/papers/AGII.pdf