UNIVERSIDAD DE **MURCIA**

FACULTY OF **MATHEMATICS**

Discrete dynamical systems

Syllabus

Course code:	6366
Number of ECTS credits:	3
Semester:	1st (September-January)
Recommended components:	Real functions of a single variable I (1568), Linear algebra (1569), Real functions of a single variable II (1573), Functions of several real variables I (1578). While strictly speaking only some standard knowledge of real analysis and linear algebra is required, the student will benefit from a background in topology and differential equations (in particular, in qualitative theory of differential equations).
Language of instruction:	Spanish (students are allowed to ask questions and write homeworks and exams in English)

Course description

In this course the student will get acquainted with some relevant topics from the theory of discrete dynamical systems. This is currently a mathematical area of special interest, both for the richness and beauty of its theoretical contents and for its usefulness in applied sciences.

In order that the course can be followed by as wide a range of students as possible, a "low-profile" syllabus has been devised, requiring only some standard background in real analysis and linear algebra. This does not mean that the contents are uninteresting; among other topics, solving of linear difference equations, global and local attraction, periodicity and chaos, and applications to models from natural sciences, will be discussed.

Learning outcomes and competences

After completion of this course you will:

1. understand the notion of dynamical system (and, in particular, that of discrete dynamical system), and related basic notions.
2. know how to model some population dynamics phenomena using discrete dynamical systems.
3. know the basic properties of linear dynamics and how to use it to understand the local behaviour of non-linear systems.
4. know the possible sets of periods of interval continuous mappings and identify the dynamical consequences of this property.

5. link the notions of chaos and sensitivity to initial conditions, and be able to compare different definitions of chaos.

6. understand the role of differentiability in the dynamics of interval mappings, and learn how to use it to check whether a local attractor is global.

7. realize the usefulness of Cantor sets in dynamics.

8. get some elementary knowledge of ergodic theory.

9. know how to use a computer to extract some relevant information from a discrete dynamical system.

Course contents

I. THEORY

1. Introduction to dynamical systems
   Basic notions: fixed points, attraction, stability and repulsion. Discrete dynamical systems. Examples. Population dynamics models.
2. Linear difference equations
   Solving method. Systems of linear difference equations. Applications.
3. Local and global attraction
   Local theory. The contractive mapping theorem. The Coppel theorem. The Singer theorem.
4. Periodicity and chaos
   The Sharkovsky theorem. Sensitivity to initial conditions. The Li-Yorke theorem. The role of Cantor sets in dynamics. Almost everywhere dynamics: an introduction to ergodic theory.

II. COMPUTER PRACTISES

1. The dynamics of the logistic family
   The logistic family is a paradigmatic one in low-dimension dynamics. Its complex nature will be elucidated with the help of an appropriate informatics tool.

References

Main texts

1. Devaney, R. *An introduction to chaotic dynamical systems*; Westview Press, 2003.

2. Salinelli, E. & Tomarelli, F. *Modelli dinamici discreti*; Springer, 2009.

Supplementary references

1. Block, L. S. & Coppel, W. A. *Dynamics in one dimension*; Springer, 1992.

2. Elaydi, R. *An introduction to difference equations, third edition*; Springer, 2004.

3. Kelley, W. G. & Peterson, A. C. *Difference equations: an introduction with applications, second edition*; Academic Press, 2001.

4. de Melo, W. & van Strien, S. *One-dimensional dynamics*; Springer, 1993.

5. Smítal, J. *On functions and functional equations*; Adam Hilger, 1988.