UNIVERSIDAD DE **MURCIA**

FACULTY OF **MATHEMATICS**

Convex and Discrete Geometry

Syllabus

Course code:	6371
Number of ECTS credits:	6
Semester:	1st (September-January)
Recommended components:	Real functions of a single variable I (1568), Linear algebra (1569), Real functions of a single variable II (1573), Functions of several real variables I (1578). While strictly speaking only some standard knowledge of real analysis and linear algebra is required, the student will benefit from a background in measure theory and basic topology.
Language of instruction:	Spanish (students are allowed to ask questions and write homeworks and exams in English)

Course description

Convex geometry dates back to antiquity. Results on this topic can already be found in the works of Archimedes, Euclid or Zenodorus (Platonic solids, isoperimetric problem, volume of pyramids...), although its main development took place during the 19th and the 20th centuries (Steiner, Cauchy, Blaschke, Minkowski, Aleksandrov and many others) and nowadays. Regarding Discrete Geometry, Kepler was the first mathematician to consider problems in this field.

The main purpose of this course is to introduce the student in the world and research of Convexity, Asymptotic Geometric Analysis and Discrete Geometry, fields which are unknown for them during the studies of Mathematics in Murcia University. Thus, the course will start *from scratch*, introducing the basic concepts and results on this topic and deepening gradually. We will study convex bodies in the Euclidean space, making use of both analytical and geometric tools. Next, the relation between the volume and the sum of sets will be considered (the Brunn-Minkowski theory), as well as some important results in Asymptotic Geometric Analysis. Finally, we will consider Geometry of Numbers, a branch of Geometry which was developed by Minkowski around 1900 in order to solve problems of Number Theory from a geometric point of view.

Learning outcomes and competences

After completion of this course you will:

1. know the main properties of the convex bodies.

2. understand the basis of the Brunn-Minkowski theory: the relation between the volume and the Minkowski addition of convex sets and the main consequences arising from it.

3. know different ways of symmetrizing sets, as well as their main properties.

4. know the basis of the Asymptotic Geometric Analysis: John's ellipsoid theorem, Banach-Mazur distance, hyperplane conjecture...
5. know and understand the fundamental theorems of Minkowski in the Geometry of Numbers and several of their many applications.

Course contents

I. Convex sets and their properties

   Convex sets. Metric projection. Support hyperplanes. Separation theorems. Polytopes. Duality. Extremal representations. Support function. Hausdorff metric and Blaschke's selection theorem.

II. The Brunn-Minkowski Theory

   Minkowski addition. Volume and surface area. Steiner formula. Quermassintegrals. Mixed volumes. Brunn-Minkowski inequality. Isoperimetric inequality. Minkowski's inequalities. Steiner, central and Schwarz symmetrizations.

III. Asymptotic Geometric Analysis

   John's ellipsoid theorem. Reverse isoperimetric inequality. Banach-Mazur distance. Almost-spherical sections. Measure concentration phenomenon on the sphere. Hyperplane conjecture.

IV. Discrete Geometry

   Lattices and bases. Mahler's selection theorem. Minkowski's Fundamental Theorem. Applications to Number Theory. Minkowski-Hlawka's Theorem. Successive minima. Minkowski's Second Theorem. Generalizations of Minkowski's theorems.

References

Main texts

1. Gruber, P. M. *Convex and Discrete Geometry*; Springer-Verlag, Berlin Heidelberg New York 2007.
2. Schneider, R. *Convex Bodies: The Brunn-Minkowski Theory*; 2nd ed., Cambridge University Press, Cambridge 2014.

Supplementary references

1. Gruber, P. M. & Wills, J. M. (eds.). *Handbook of Convex Geometry*; North-Holland, Amsterdam 1993.
2. Gruber, P. M. & Lekkerkerker, C. G. *Geometry of numbers*; North-Holland, Amsterdam 1987.
3. Lay, S. R. *Convex Sets and their Applications*; Wiley, New York 1982.
4. Pisier, G. *The volume of convex bodies and Banach space geometry*; Cambridge University Press, Cambridge 1989.
5. Webster, R. *Convexity*; Clarendon Press Oxford University Press, New York 1994.