S8 Quantum Technologies Tecnologías Cuánticas (GEIC-GEOCONL)

13/07 Wednesday afternoon, Aula 1.11

15:30-16:15	Carlos Navarrete Benlloch Invited Talk
16:15-16:30	Xi Chen Variational Approach to Coherent Matter-Wave Splitting
16:30-16:45	Yan Chen, Yue Ban, Ran He, Jin-Ming Cui, Yun-Feng Huang, Chuan-Feng Li, Guang-Can Guo, and Jorge Casanova A Neural Network Assisted ¹⁷¹ Yb ⁺ Quantum Magnetometer
16:45-17:00	Nilo Mata, Isabel Gonzalo, Miguel A. Porras Promoting quantum tunneling via Zeno Dynamics
17:00-18:00	Coffee Break
18:00-18:15	Miguel Varga, Jon Lasa-Alonso, Carlos Maciel-Escudero, Martín Molezuelas-Ferreras, Iker Gomez-Viloria, Garbriel Molina-Terriza Understanding the normal modes of optical fibers for quantum communications
18:15-18:30	Álvaro Navarrete, Marcos Curty Decoy-State Quantum Key Distribution Secure Against Trojan-Horse Attacks
18:30-18:45	Víctor Zapatero, Marcos Curty Simple and passive quantum key distribution
18:45-19:00	Xoel Sixto, Víctor Zapatero, Álvaro Navarrete, Marcos Curty <i>Quantum cryptography with intensity correlations</i>
19:00-19:15	Cristian Tabares and Alejandro González-Tudela Tunable photon-mediated interactions between spin 1 atoms
19:15-19:30	Santiago Oviedo Casado Power-law scaling of correlations in statistically polarized nano-NMR

Time-crystalline order in lone and coupled quantum self-sustained oscillators

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The possibility of observing systems in a state of spontaneous robust temporal oscillations insensitive to the effect of any source of noise (including the unavoidable quantum fluctuations) has sparked a lot of debate in the last decade. Originally introduced as potential exotic equilibrium states of matter [1], these were dubbed *time crystals* by analogy with regular crystals, whose periodic spatial pattern shows the same type of robustness. While nowadays we know that time-crystalline order cannot exist in generic systems at equilibrium, several non-equilibrium workarounds have been suggested to circumvent their limitations. In particular, it has been suggested that the long-term (asymptotic) state of driven-dissipative systems might spontaneously break time-translational symmetry, given the right conditions.

In this work we consider the quantum version of an incoherently-pumped oscillator subject to nonlinear loss (so-called, *quantum Van der Pol oscillator* [2]), which in the classical regime serves as the paradigm of a self-sustained oscillator. We use this generic driven-dissipative model to analyze how robust the classical oscillatory states are to quantum noise, arriving to the conclusion that quantum fluctuations always tend to restore (through phase diffusion) the time-translational symmetry broken at the classical level. In this sense, we argue that such systems are not true time crystals, since their oscillations are not robust. We prove this both through a Floquet-based linearization technique that we developed recently [2,3], as well as first-principles numerical simulations based on phase-space stochastic techniques.

In an attempt to halt the phase diffusion responsible for destroying time-crystalline order, we consider a feedback mechanism by which the oscillator's state at some time t acts as a driving mechanism at a later time $t+\tau$. Denoting by g the feedback rate strength, we show that in this case the phase diffusion rate receives a $(1+g\tau)^{-2}$ contribution, which indeed makes the oscillations more robust, but still not fully insensitive to quantum fluctuations except in the unphysical $g\tau \to \infty$ limit.

On the other hand, we also study the possibility of halting phase diffusion and stabilize the self-sustained oscillatory states by coupling several oscillators in a lattice, entering the many-body realm. Let us denote by *L* the number of oscillators (system size), which we arrange in 1D, 2D, or 3D square geometries, but consider as well all-to-all connections, that is, the so-called mean-field or ∞D limit. Our analysis shows that the phase diffusion rate receives now a 1/L contribution at long times for all geometries, and hence one of the conditions for the existence of time-crystalline order is indeed satisfied: many-body effects in the thermodynamic $L \rightarrow \infty$ limit halt phase diffusion asymptotically. However, we also show that by the time phase diffusion is halted, the phase variance has reached an asymptotic value that scales as L^{β} , where $\beta = 0$ for ∞D , $1>\beta>0$ for 2D and 3D, and $\beta>1$ for 1D. This is a consequence of the Lieb-Robinson bound, which limits the speed at which correlations can propagate in the many-body system, and which applied to our case says that time-crystalline order is clearly achieved for ∞D and might be achieved in practice for large-but-finite systems in 2D and 3D, but possibly not at all in 1D. To our knowledge, it is the first time that this strong conclusion is unequivocally reached by considering full quantum dynamics of a driven-dissipative many-body lattice model.

We propose concrete implementations based on modern quantum platforms such as superconducting circuits where our findings can be experimentally explored.

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Variational Approach to Coherent Matter-Wave Splitting

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We report an efficient protocol for coherent matter-wave splitting in Bose-Einstein condensates. Following the variational principle [1], we inversely engineer the quantum dynamics in the mean-field approximation, which is indeed the shortcuts to adiabaticity [2] for the system. We evaluate the shortcuts by numerical simulations with the multi-configurational time-dependent Hartree method for bosons, see Fig. 1. We also introduce atom interferometry as one of the most straightforward applications of our protocol. Further extensions of the work will pave the way for developing quantum technologies for metrology.



Figure 1. (a) The initial ground state of the BEC with nonlinearity gN=1 (curve) and the variational ansatz (dotted curve). The fidelity between them is F=0.99755, where the optimal parameter a=1.05. (b) The wave packets after coherent splitting, the target ground state (dashed curve), and the variational ansatz (dotted curve). We simulate the many-body dynamics by MCTDHB of M=3 with N=1000 bosons. The fidelity between the final state and the target state is F=0.99996. (c) The wave packet dynamics during the STA of operation time t_f=7.60. We plot the heatmap of $|\Psi(x,t)|^2$. The BECs population on the three natural orbits are 999.59, 0.40704, and 0.0013845, respectively.

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A Neural Network Assisted ¹⁷¹Yb⁺ Quantum Magnetometer

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A versatile magnetometer must deliver a readable response when exposed to targets fields in a wide range of parameters. In this work, we experimentally demonstrate that the combination of a ¹⁷¹Yb⁺ atomic sensor with adequately trained neural networks [1] enables the characterisation of target fields in distinct challenging scenarios, schematically shown in Fig. 1. In particular, we estimate parameters of radio frequency drivings with large shot noise, including the limiting case of continuous data acquisition with single-shot measurements, and in regimes where the magnetometer delivers a response that significantly departs from the standard harmonic behavior [2].



Figure 1. (a) Energy scheme of the 171 Yb⁺ atomic sensor and the corresponding transitions. This leads to the level configuration in (b) which is defined in the dressed state basis. (c) Schematic configuration of the experimental setup. (d) Scheme of the neural network. The response from PMT, i.e. **X**, is processed by a number of hidden layers and the NN gives the outputs **Y**.

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Promoting quantum tunneling via Zeno Dynamics

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Quantum tunneling (QT) is one of the most fundamental phenomena in quantum physics. It plays a crucial role in many physical processes such as nuclear fusion, high-energy particle collisions, reaction kinetics, and is at the core of recent quantum technologies [1]. On the other hand, and unlike classical objects, a quantum system is perturbed under the action of measurements. In particular, it is known that frequently checking whether a quantum system is in its initial state slows down the evolution [2, 3], which is called quantum Zeno effect, and more generally quantum Zeno dynamics (QZD). The higher the frequency of measurements, the lower the probability of evolution, which is impossible with continuous measurements [2, 3].

Mixing QT and QZD, we study the dynamics of a tunneling particle while it is frequently observed [4]. Unlike previous works, ours involves momentum measurements. These are performed to ascertain if the momentum direction is the same as initially, say, positive. Thus, we prevent the natural evolution of the particle (reflection changing its momentum sign), as in Fig. 1(a), forcing the particle to mantain its initial positive sign, as in Fig. 1(b), and causing the particle to traverse the barrier easier. Tunneling probability P_N increases with the number N of measurements within the tunneling time interval, as seen in Fig. 1(c), and tends to unity when N goes to infinity. Measurements are less effective as the particle becomes classical (higher action), as illustrated in Fig. 1(d).



Figure 1. a) Initial (dashed), partially reflected and transmitted (red) probability densities through a potential barrier (dotted). Position is in units of the initial wavepacket width Δx . b) The same but with N measurements. c) Tunneling probability versus number of measurements. d) Tunneling probability versus number of measurements and normalized action $p_0 \Delta x/\hbar$, where p_0 is the initial mean momentum of the particle.

We model the measurements as von Neumann projections into the positive-momentum subspace (selective), and also as short interactions with probe particles (nonselective) that acquire information on the sign of momentum. With both models of measurements QZD enhances QT, nonselective measurements being more effective.

The implementation of this QZD control of QT would have an impact on the quantum technologies where QT is a key ingredient.

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Decoy-State Quantum Key Distribution Secure Against Trojan-Horse Attacks

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Most security proofs of quantum key distribution (QKD) disregard the effect of information leakage from the users' devices, and, thus, do not protect against Trojan-horse attacks (THAs). In a THA, the eavesdropper injects strong light into the QKD apparatuses, and then analyzes the back-reflected light to learn information about their internal setting choices. Only a few recent works consider this security threat, but predict a rather poor performance of QKD unless the devices are strongly isolated from the channel.

We present an improved finite-key security analysis for decoy-state-based QKD schemes in the presence of THAs, which significantly outperform previous analyses [1]. For this, we take advantage of the reference technique [2] equipped with a Cauchy-Schwarz-based constraint to incorporate the information leakage from the bit/basis and intensity encoding setups in the security analysis. This requires the users to bound a single parameter that encapsulates all the imperfections, which in practice can be directly related to the amount of isolation of Alice's transmitter. Besides, we use novel concentration bounds to deal with the finite-key effects.

For illustration purposes, we evaluate the performance of the standard decoy-state BB84 protocol and the decoy-state loss-tolerant protocol in the presence of THAs. The results demonstrate the feasibility of both schemes over long distances given that the information leakage is small enough, which could be achieved by increasing the isolation of the devices. Besides, for the decoy-state BB84 case, we compare the secret-key rate attainable with our security proof with that obtained in previous works [1]. The improvement in terms of performance is clear, as shown in Fig. 1, in which our analysis basically allows to double the maximum achievable distance attained by previous works in some realistic scenarios.



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Simple and passive quantum key distribution

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Quantum key distribution (QKD) exploits fundamental properties of quantum-mechanical systems to securely distribute cryptographic keys between remote parties through an insecure channel. This task is known to be impossible using classical cryptography alone, which makes QKD a promising candidate for long-term communication security. Nowadays, QKD has become a mature and prolific industry within the field of quantum information science, thanks to the combined efforts of theorists and experimentalists. Notwithstanding, the security of real QKD implementations is not fully established yet, due to the difficulty of experimentally guaranteeing that the QKD devices stick to the assumptions and models presumed in the security proofs.

A particularly controversial assumption present in QKD security analyses is that no information leakage takes place through the boundaries of Alice's and Bob's labs. This premise opens the door for the so-called Trojan horse attacks (THAs), where an adversary injects bright light pulses into a QKD transmitter and then measures the back-reflected light, aiming to extract information about the internal setting choices of the transmitter. A possible solution to avoid this problem is to consider a passive (rather than active) QKD transmitter, where the protocol settings are set at random using inherent quantum randomness of the device, in so circumventing THAs. What is more, hardware-wise, passive transmission typically allows simpler architectures than active transmission. Notably though, these advantages come at the price of decreasing the key generation rate. This is so because, in a passive transmitter, additional post-selection is required to discard those rounds where the randomly generated settings do not lie in certain acceptance intervals.

In this work, we propose a simple and passive QKD transmitter tailored for the decoy-state BB84 protocol [1,2,3], the most popular practical QKD protocol to the date. Essentially, our scheme carefully combines two previously-known ideas: a proposal to passively generate random photon-polarizations in a plane (suitable for the BB84) [4], and a proposal to passively generate random intensity laser pulses for the application of the decoy-state method [5]. On top of it, we elaborate a custom-made security analysis to evaluate the secret-key-rate performance of our passive QKD transmitter.

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Quantum cryptography with intensity correlations

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Quantum key distribution (QKD) enables information-theoretically secure delivery of cryptographic keys between distant parties through an insecure channel. This possibility (which is inaccessible from the point of view of classical communications) makes QKD a promising candidate for private communications secure in the long-term. For this reason, QKD has experienced a tremendous development both in theory and in practice, in so becoming the most mature application of quantum information science. Nevertheless, despite this progress, various challenges must be addressed in order to achieve the widespread adoption of QKD.

In particular, aiming to enhance the key generation performance of QKD implementations, it is very desirable to boost the repetition rate of the laser sources in QKD transmitters. Nevertheless, for clock rates of the order of GHz, intensity correlations between succeeding pulses appear [1,2], possibly opening a security loophole. To be precise, in the absence of ideal single-photon sources, most QKD protocols use attenuated lasers that operate emitting phase-randomised weak coherent pulses. These sources allow QKD users to implement the so-called decoy-state method [3,4,5], a standard procedure that enables a tight estimation of the secret key length of a QKD session by means of using multiple intensities for QKD transmission. Unfortunately though, the presence of intensity correlations invalidates the central assumption of the decoy-state method, namely, that the photon-number detection statistics do not depend on the intensity of the laser. This being the case, the question arises of how to account for arbitrary intensity correlations in the security analyses of QKD, a question for which existing results are notably restricted [1,6,7].

In two consecutive works [8,9], we solve this pressing problem by developing the missing security analysis for decoy-state QKD with arbitrary intensity correlations. The central ideal is to pose a restriction on the maximum bias that an adversary may induce between the photon-number detection statistics associated to different intensity settings. Particularly, this is accomplished by using a fundamental result presented in [10], which constraints how much the measurement statistics of non-orthogonal quantum states can differ. We refer to this result as the Cauchy-Schwarz constraint, because it is a consequence of the Cauchy-Schwarz inequality in complex Hilbert spaces. Notably, our analysis relies on a very general and experimental-friendly characterization of the intensity correlations. Moreover, it may be combined with detailed correlation models, which are expected to be pointed out by ultrafast QKD experiments in the near future. Putting it all together, our results provide a fundamental step towards foolproof security of high-speed QKD systems, with their imperfections.

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Tunable photon-mediated interactions between spin 1 atoms

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Quantum simulators are highly controllable devices that exploit quantum effects to answer questions about another system. They can be built using different platforms, such as ultracold atoms in optical lattices, superconducting circuits or atoms interacting with nanophotonic structures [1]. This last system is particularly interesting because the nanophotonic environment can be tailored to generate exotic photon-mediated interactions between atoms [2], with both dissipative and coherent evolutions, opening the door for the exploration of a wide range of physical models. However, these atoms have been typically considered as two-level systems, which limits the type of models that can be explored [3,4]. Our work considers the full hyperfine structure of the atoms to go beyond this and study effective spin-1 interactions between the quantum emitters, where Raman-assisted transitions allow a mapping to well known models such as the Ising or the XX spin-1 interactions. In this context new complications appear, such as the different rates of each transition according to the corresponding Clebsch-Gordan coefficients, and we also propose different ways to solve them. These results could be interesting both in quantum simulation (where they could be applied to study spin chains or even simulating some lattice gauge theories [5,6], and also be the base for digital-analog schemes) and quantum computation (as a way to obtain quantum gates between qutrits [7]).



Figure 1. Main panel: Schematic representation of two atoms trapped near a 1D photonic crystal waveguide, showing the atom-photon bound state that allows the tunable interactions. Inset: Full multilevel structure of a real atom, including the excited states that are adiabatically eliminated to obtain the effective spin-1 interactions between the quantum emitters.

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