An Application of Uncertain Variables to the Implementation of an Interacting Multi-Model based Filter for the Navigation System of a Road Vehicle

Rafael Toledo-Moreo, Miguel A. Zamora-Izquierdo and Antonio F. Gómez-Skarmeta

***Abstract*— This paper is to present the conclusions obtained from the parametric study of the application of the uncertain variable (UV) theory to the problem of the interactive multi-model method (IMM) in the navigation system of a road vehicle. The proposed navigation system implements an interactive multi-model method based on a loosely coupled extended Kalman filter architecture (EKF). The descriptions of the navigation system and the representation of the problem based on uncertain variables are presented in this paper. Selected simulations and their conclusions are commented. Finally, this parametric study is presented as a helpful tool for the optimization of an IMM based data fusion algorithm for navigation systems in road vehicles.**

I. Introduction

It is well known the importance of the vehicle model definition in the development of navigation systems for road vehicles. However, it is not easy to find a proper unique model suitable in any situation. In the recent years, several authors have been centering their efforts in models capable to combine the information coming from the odometry of the vehicle, inertial sensors and satellite navigation [1]–[8].

The implementation of interactive multi-model (IMM) based methods allows the possibility of using highly dynamic models just when required, diminishing unrealistic noise considerations (in non maneuvering situations) and the computational charge of the system. Some solutions based on this approach can be seen in [9], [10]. As can be seen in [11], the combination of IMM techniques with different probabilistic data association methods has improved the single IMM results in the aerial navigation field. Recently, some authors like Huang and Leung in [12] have applied those techniques to the road field, obtaining interesting results of their simulations.

The solution presented in this paper is based on the use of the uncertain variable theory in the decision making problem of the vehicle model in an interactive multi-model based method. As can be seen in [13], [14], different approaches based on knowledge representation with unknown parameters have been applied to the Intelligent Transport System (ITS) problems in the last years. In this field, the theory of the uncertain variables (UV) has been proved as a useful tool for the optimization process, like presented in the works [15]–[19]. In [19], examples of the application of the uncertain variable theory to several simple problems can be found. This interesting book analyzes the advantages of using the uncertain variables in the optimization process of the decision making problem, taking into account the user constraints, and describing the system in a relational knowledge representation by equation and inequations with unknown parameters.

In order to apply the UV to the problem of the optimization of data fusion in an interactive multi-model based method, certain considerations regarding its knowledge representation must be done. In this work, the uncertain variables are used as estimators of the suitability of two proposed kinematical models of the vehicle. In this sense, the models are understood as experts with respective certainty distributions associated to their estimates. Since the aim of this work is not to realize a complete study of the uncertain variables but its application to our specific problem, the number of possible distributions supplied by the experts has been reduced. Additionally, only one knowledge representation of the relation between the defined uncertain variables and the kinematical observations is considered.

Along this paper, firstly, the description of the proposed navigation system is presented, including the vehicle and the sensor models and the interactive multi-model bases. Secondly, the description of the uncertain variable representation is explained. Finally, selected simulations of the parametric study and their conclusions are commented.

II. The Navigation System Implementation

The navigation system proposed in this paper is based on a GNSS/SBAS/INS integrated system. On one hand, the use of combined GNSS/SBAS (Satellite Based Augmentation Systems), as compared with the single GPS solution, provides noticeable improvements, but nevertheless, they cannot fulfill the requirements of high integrity demanding applications, specially in city environments. On the other hand, the INS (Inertial Navigation System) units supply accelerations and rates of turn relative to the three Cartesian axis of the sensor body frame. Although these measurements complement the GNSS/SBAS lacks and provide positioning during the outages of the satellite signal, the double integration process required to obtain position from the acceleration is the main source of error for the INS units. In order to avoid excessive drifts, often updates must be performed by a global system. In addition, only low cost inertial units, based on micro-electro-mechanical (MEM) technology, are affordable

Rafael Toledo-Moreo, Miguel A. Zamora-Izquierdo and Antonio F. Gómez-Skarmeta are with Department of Information and Communication Engineering, Faculty of Computer Science, University of Murcia, 30071 Murcia, Spain toledo@um.es

considering a real mass market road side equipment (RSE). Unfortunately, these sensors present bad noise features and drifts and the implementation of error models is advisable. In order to diminish the drifts during the GNSS outages, odometry measurements coming from the ABS (Anti-Blocking System) encoders of the vehicle are also considered in our system. The ABS system provides non precise velocity information, with a very low increase of the final cost, since no further installations or sensors are needed. Apart from the precision problem due to the low level of performance of the ABS encoders, typical odometry problems, such as glides, unequal wheel diameters or effective wheel diameter uncertainty are also presented.

To obtain the proper inputs to the data fusion filter from the raw measurements coming from the sensors, observation models are implemented, and considerations about the sensor performances done.

A. The Multisensor Data Fusion Filter

The data fusion filter developed to combine the information coming from the GNSS, INS and odometry sensors is based on a loosely coupled extended Kalman filter architecture, implementing an interactive multi-model method to employ the vehicle model definition which better describes the current vehicle's behavior.

1) The Kalman Filter: The Kalman filter is a recursive least squares estimator. It produces at time k a minimum mean squared error estimate $\hat{\mathbf{x}}(k|k)$ of a state vector $\mathbf{x}(k)$. This estimate is obtained by fusing a state estimate prediction $\hat{\mathbf{x}}(k|k-1)$ with an observation $\mathbf{z}(k)$ of the state vector $\mathbf{x}(k)$. The estimate $\hat{\mathbf{x}}(k|k)$ is the conditional mean of $\mathbf{x}(k)$ given all observations $\mathbf{Z}^k = [\mathbf{z}(1), \cdots \mathbf{z}(k)]$ up until time k,

$$\hat{\mathbf{x}}(k|k) = \mathbf{E}[\mathbf{x}|\mathbf{Z}^k] \tag{1}$$

where $\mathbf{Z}^k$ is the sequence of all observations up until time k.

2) The Vehicle Models: In order to represent the movements of the vehicle along roads, two models have been developed. Both are based on the rigid solid definition of a four wheel vehicle, the back wheels of which can rotate only about a transversal axis of the vehicle, and the forward wheels turn describing curves centered in their instant rotation center. The straight model (or non-maneuvering model) represents a basic non-maneuvering behavior of the vehicle, being its transition equation defined as:

$$\mathbf{x}(k+1) = f(\mathbf{x}(k)) + G(\mathbf{x}(k))\upsilon(k) \tag{2}$$

where f is the state transition matrix and G the noise matrix, and the state and noise vectors are respectively:

$$\begin{aligned} \mathbf{x}(k) &= [x_c(k)\ y_c(k)\ \theta(k)\ \dot{\theta}(k)\ v_c(k)\ \phi_c(k)\ s_c(k)]^T \\ \upsilon(k) &= [\ddot{\theta}(k)\ \dot{v}_c(k)\ \dot{\phi}_c(k)\ \dot{s}_c(k)]^T \end{aligned} \tag{3}$$

where $x_c(k)$, $y_c(k)$ are the coordinates of the geometrical centre of the vehicle (g.c.), $\theta(k)$ the vehicle orientation, $v_c(k)$ the velocity in the g.c., $\phi_c(k)$ is the angle of the velocity $v_c(k)$, and $s_c(k)$ the slide correction angle. In the straight model, $\phi_c(k)$ is modelled by a first order function, so both straight and mild trajectories fulfill the kinematical definition of the model. However, when sharp curves are performed, it is advisable to represent $\phi_c(k)$ by a second order equation. Thus, the state and noise vectors of the curved model (or maneuvering model) are:

$$\begin{aligned} \mathbf{x}(k) &= [x_c(k)\ y_c(k)\ \theta(k)\ \dot{\theta}(k)\ v_c(k)\ \phi_c(k)\ \dot{\phi}_c(k)\ s_c(k)]^T \\ \upsilon(k) &= [\ddot{\theta}(k)\ \dot{v}_c(k)\ \ddot{\phi}_c(k)\ \dot{s}_c(k)]^T \end{aligned} \tag{4}$$

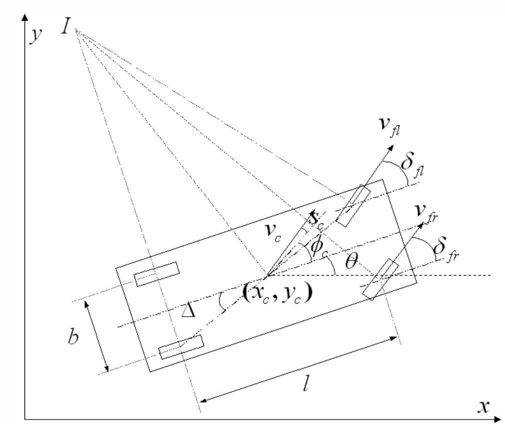

Fig. 1. The kinematical model nomenclature.

Fig. 1 shows graphically the kinematical model and its nomenclature.

3) The Sensor Models: In order to obtain the filter observations $\mathbf{z}(k)$ at the scan k, different transformations must be done. The observation vector of our system is defined as:

$$\begin{aligned} \mathbf{z}(k) = & [x_c^G(k)\ y_c^G(k)\ x_c^I(k)\ y_c^I(k) \\ & \theta^I(k)\ \dot{\theta}^O(k)\ v_c^O(k)\ \phi_c^O(k)\ v_c^I(k)]^T \end{aligned} \tag{5}$$

where $x_c^G(k)$, $y_c^G(k)$ and $x_c^I(k)$, $y_c^I(k)$ are the Cartesian coordinates of the g.c. according to the GNSS and the INS measurements respectively, $\theta^I(k)$ and $v_c^I(k)$, obtained from the inertial measurements, define severally the orientation and the velocity of the g.c. of the vehicle and finally, $\dot{\theta}^O(k)$, $v_c^O(k)$ and $\phi_c^O(k)$ are respectively the angular velocity, the linear velocity and its angle in the g.c. observed by the odometry system. Some of these variables can be easily obtained from the sensor measurements. In this section we present the transformations required to obtain the observations $\dot{\theta}^O(k)$, $v_c^O(k)$, $\phi_c^O(k)$, $x_c^I(k)$, $y_c^I(k)$ and $v_c^I(k)$.

4) The Odometry Observations: Regarding the odometry observations, taking into account the assumption of the vehicle as a rigid solid, the velocity in the g.c. $v_c(k)$ can be calculated as:

$$v_c^O(k) = v_{fl}(k)\frac{\cos(\Delta - \delta_{fl}(k))}{\cos(\Delta - \phi_c(k) - s_c(k))} \tag{6}$$

where $v_{fl}(k)$ and $\delta_{fl}(k)$ are respectively the velocity and the angle of the forward left wheel. The angular velocity can be calculated depending on these variables as:

$$\dot{\theta}^O(k) = v_{fl}(k)\frac{\sin(\delta_{fl}(k))}{l} \quad (7)$$

Finally, to calculate the angle of the velocity in the g.c. (fig. 1), the geometrical transformations from the angles of the forward wheels to the g.c. are given by:

$$d = \frac{l}{\tan(\delta_{fl})} + \frac{b}{2}$$
$$\tan(\phi_c(k) + s_c(k)) = \frac{l/2}{d} \quad (8)$$

Thus, the angle of the velocity is:

$$\phi_c^O(k) = \arctan\left(\frac{l \cdot \tan(\delta_{fl}(k))}{2l + b \cdot \tan(\delta_{fl}(k))}\right) - s_c(k) \quad (9)$$

5) The Inertial Observations: To obtain the inertial observations four different phases must be performed:

Whereas low cost inertial sensors are used, error models must be considered. The first step must be the implementation of error models for the inertial measurements. The models implemented in our work are based on the Billur Barshan work [20], and can be described by the expression:

$$\varepsilon = C_1\left(1 - e^{\frac{-t}{\tau}}\right) + C_2 \quad (10)$$

where ε represents the error model for the acceleration in the body frame of the sensor and C_1, C_2 and τ are model parameters. Fixing the values $C_1 = -0.0043$, $C_2 = -0.007$ and $\tau = 500$ by using a Nelder-Mead non-restricted non-linear multidimensional method where the minimizing function was the mean squared error, a mean value of -3.2172×10^{-4} and a standard deviation value of 0.0033 were achieved for the compensation of the forward acceleration of the body frame. With these values, in tests where no forces were applied to the sensors (but the Earth gravity) and no external updates were performed, the position drifted 70 cm. after 60 seconds. In the same tests, but without applying any error model, the position drifted up to 55 m.

Secondly, in order to obtain the acceleration vector referenced to the global frame (North-East-Down) (**G**) from the local reference (**S**), the rotation matrix ${}^{GS}\mathbf{R}$ defined in [10] can be used.

Then, a gravitational model must be applied to compensate the Earth gravity effects. Typically, in terrestrial applications with mobile units, the gravity is assumed to value $-9.81\ \mathrm{m/s^2}$. in the z axis of the global reference frame (local tangent plane).

As a final step, the inertial observations x_c^I and y_c^I can be calculated by applying the equation

$$x_c^I(k+1) = x_c(k) + v_{c_x}(k)T + 0.5 \cdot a_x T^2 \quad (11)$$
$$y_c^I(k+1) = y_c(k) + v_{c_y}(k)T + 0.5 \cdot a_y T^2 \quad (12)$$

where $x_c(k)$ and $y_c(k)$ are the state variables just after the last update, T is the difference between the time stamp of the inertial measurements and the time stamp of the last measurement which updated the state vector, a_x and a_y are the acceleration values in the global reference system, as obtained from the previous step, and the values of the velocities v_{c_x} and v_{c_y} are given by the equations

$$v_{c_x}(k) = v_c(k)\cos\left(\theta(k) + \phi_c(k) + s_c(k)\right) \quad (13)$$
$$v_{c_y}(k) = v_c(k)\sin\left(\theta(k) + \phi_c(k) + s_c(k)\right) \quad (14)$$

The observation v_c^I can be calculated by using the expression

$$v_c^I(k+1) = v_c(k) + a_t^I T \quad (15)$$

where $v_c(k)$ is the state variable just after the last update and a_t^I represents the module of the acceleration tangential to the vehicle's trajectory, calculated according to the inertial measurements. To calculate a_t^I, we will assume that the geometrical and the gravity center of the vehicle coincide in (x_c, y_c). Naming α the angle between the absolute acceleration vector of the vehicle, **a**, and the x axis, we can affirm that

$$\alpha = \arccos\left(\frac{a_x}{a}\right) \quad , \quad a = \sqrt{a_x^2 + a_y^2} \quad (16)$$

where a_x, a_y are the horizontal components of the vector **a** and a its projection on the xy plane. Besides, the module of the tangential acceleration can be calculated as

$$a_t^I = a\cos\left(\alpha - (\theta + \phi_c + s_c)\right) \quad (17)$$

Thus, next expression for the a_t^I value can be obtained

$$a_t^I = \sqrt{a_x^2 + a_y^2} \cdot \cos\left(\arccos\left(\frac{a_x}{a}\right) - \left(\theta(k) + \phi_c(k) + s_c(k)\right)\right) \quad (18)$$

B. The EKF Implementation

The implementation of the EKF is developed in three phases: prediction and observation of the state and its covariance $\hat{\mathbf{x}}(k|k-1)$, $P(k|k-1)$, calculation and validation of the observation innovations coming from the sensors $\nu(k)$, and calculation of the Kalman gain $W(k)$ and update of the state and its covariance $\hat{\mathbf{x}}(k|k)$, $P(k|k)$. More details on the Kalman implementation can be found in [10].

C. The IMM Filter

In most of the real driving situations it is not possible to know in advance which kind of maneuvers will be performed, and the idea of selecting routes with only mild maneuvers is not very realistic. Therefore, an interactive multi-model filter has been developed and implemented. The IMM filter calculates the probability of success of each model at every filter execution scan, supplying a realistic combined solution for the vehicle's behavior. These probabilities are calculated according to a Markov model for the transition between maneuver states. The likelihood calculation and the model probability update are performed according to the statistical distance value, given by

$$d^2 = \nu^T S^{-1} \nu \quad (19)$$

Given an IMM approach, there will be a different residual covariance matrix, $S_i(k)$ and distance $d_i^2(k)$ associated with each of the i models, for the update at the scan k. Assuming measurement dimensionality M, and Gaussian statistics, the likelihood function for the observation model given model i is

$$\Lambda_i(k) = \frac{exp[-d_i^2(k)/2]}{\sqrt{(2\Pi)^M |S_i(k)|}} \tag{20}$$

Finally, using Bayes's rule, the updated model probabilities become

$$\mu_i(k) = \Lambda_i(k) C_i(k-1)/C \tag{21}$$

where the normalization constant C and C_i, the probability after interaction that the vehicle is in state i, can be calculated as described in [10].

III. The UV-IMM Approach

The uncertain variable based interactive multi-model method, UV-IMM, deals with the decision making problem of which model reproduces in a most suitable way the vehicle behaviour anytime, on the basis of the uncertain varible theory. Let us introduce the following notation:

- $\hat{e}$ is the observation variable.
- x_1, x_2 represent the statistical distances calculated by applying (19).
- x_1^*, x_2^* are the certainty distribution parameters of x_1, x_2.
- u_1 is the probability that the model 1 is the optimum (analogous for u_2), and $u_1 + u_2$ must sum to unity.
- B_1, B_2 are specified by the user, and define the validation thresholds of the models (typically, times of standard deviations). B is the maximum value adjusted to the maximum value of B_1 and B_2.

The problem consists of the determination of u_1, u_2 considering B and knowing that

$$B_1 \leq x_1 u_1 \quad , \quad B_2 \leq x_2 u_2 \tag{22}$$

According to (22), the relation between the variables x_1 and x_2 can be easily understood, paying attention to the definitions of x_1, x_2 and u_1, u_2. Equation (22) defines the probability that a model is currently the most suitable option, as inversely proportional to the statistical distance value of this model estimate to the observation solution. We assume that x_1, x_2 are unknown values of uncertain variables $\bar{x}_1, \bar{x}_2$ characterized by certainty distributions $h_1(x_1), h_2(x_2)$ given by experts. Many different relations could be established both for the representation of the relation between the uncertain variables and the model parameters of the system, and for the calculation of the defined distance between the model estimates and the observation.

Now, we can formulate the following decision making problem: find the distribution (u_1, u_2) maximizing the certainty index that the user's requirement $B(x_1, x_2) \leq \alpha$ (where $B = max\{B_1, B_2\}$) is approximately satisfied, i.e. the certainty index $v[B(\bar{x}_1, \bar{x}_2) \lessapprox \alpha]$.

We define the certainty distribution given by an expert according to the Fig. 2 (analogous for x_2).

To determine the distribution $u_1 \equiv u_1^*, u_2 \equiv u_2^*$, maximizing the certainty index v, veryfing $B \leq \alpha$ for given $\alpha > 0$, the next assumption is considered:

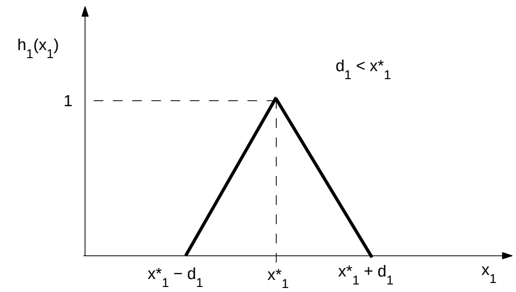

Fig. 2. Triangular certainty distribution offered by the expert.

$$\frac{x_1^*}{d_1} = \frac{x_2^*}{d_2} \equiv \sigma. \tag{23}$$

It may be shown that for

$$\frac{u \cdot (\sigma - 1) \cdot x_1^* \cdot x_2^*}{\sigma \cdot (x_1^* + x_2^*)} \leq \alpha \leq \frac{u \cdot x_1^* \cdot x_2^*}{(x_1^* + x_2^*)} \tag{24}$$

the result is:

$$u_1^* = \frac{u \cdot x_2^*}{x_1^* + x_2^*} \quad , \quad u_2^* = \frac{u \cdot x_1^*}{x_1^* + x_2^*} \tag{25}$$

and the certainty index

$$v = \sigma \cdot \left[\frac{\alpha \cdot (x_1^* + x_2^*)}{u \cdot x_1^* \cdot x_2^*} - 1 \right] + 1. \tag{26}$$

Analogously, for a parabolic distribution, the certainty index will be

$$v = 1 - \sigma^2 \cdot \left[\frac{\alpha \cdot (x_1^* + x_2^*)}{x_1^* \cdot x_2^*} - 1 \right]^2 . \tag{27}$$

IV. Results of the Simulation

The determination of the proper decision requires a very complicated optimization problem that implies numerical methods. To design a proper algorithm able to select the proper model in an optimal way, it is necessary to investigate the properties of the relationships between the uncertain parameters and the certainty index of the solution, which is the main objective of this work. Selected examples of the results obtained in our simulations are shown in this paper. Particularly, the results presented are:

- the relationship between the certainty index of the solution and the parameters x_1^*, x_2^* in the certainty distributions,
- the influence of the parameter σ of the certainty distributions on the certainty index of the expert solution,
- the impact of the expert estimation on the final result.

Figs. 3 and 4 illustrate some examples of relationships obtained as a result of our simulations.

In the investigation of the relationship between the certainty index v and the variables x_1^*, x_2^* (Fig. 3), the values of u, α and σ are fixed to 1, 0.5 and 3 respectively. For a fixed value of x_2^* , the parameter x_1^* has a significant influence on the certainty index v. For example, for $u = 8$, $\sigma = 2$,

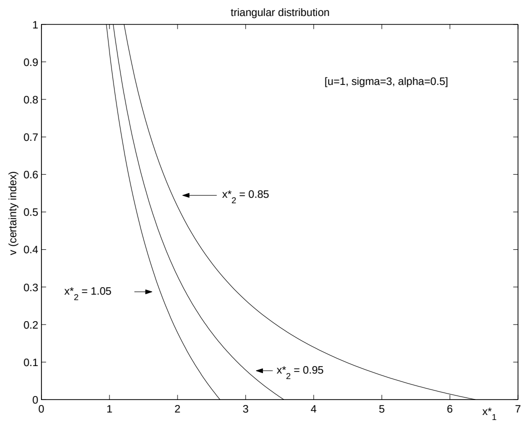

Fig. 3. Relation between x_1^* and x_2^* fixing u, α and σ in a triangular distribution.

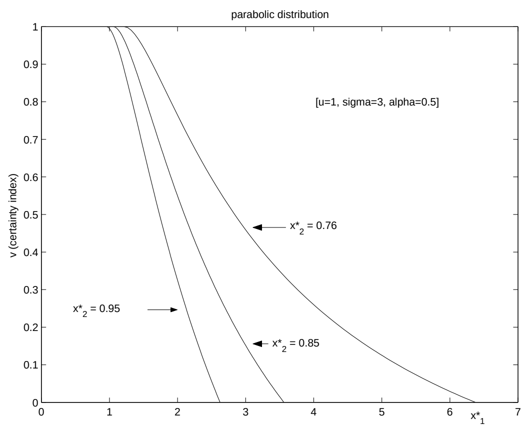

Fig. 4. Relation between x_1^* and x_2^* fixing u, α and σ in a parabolic distribution.

$\alpha = 12$, $x_1^* = 3.1$, $x_2^* = 3$, the certainty index for a triangular distribution calculated by (26) is $\nu = 0.9516$. However, for the same distribution and $u = 8$, $\sigma = 2$, $\alpha = 12$, $x_1^* = 3.2$, x_2^*, the certainty index $\nu = 0.9062$. It can be seen that, since ν decreases when x_1^* is increased, the condition of $B \leq \alpha$, will be satisfied with a higher certainty index when x_1^* is closer to the value of x_2^* (analogous results are obtained for fixed x_1^* and different values of x_2^*). This means that, when the distances observation-estimates for every model are similar, the certainty of satisfying the requirement is higher.

The sensitivity of the certainty index ν with respect to the expert estimation can be observed in the Figs. 5 and 6.

In Fig. 5, a comparison between the triangular and a parabolic distribution is shown. It is observed how the parabolic distribution offers higher certainty index for fixed u, σ, α, x_2^*, through all the values of x_1^* satisfying (24). Concretely, for $u = 1$, $\sigma = 3$, $\alpha = 0.5$, $x_1^* = 2$, $x_2^* = 0.85$, the certainty index obtained considering the triangular distribution, and calculated by (26) is $\nu = 0.5147$. For the same values but considering a parabolic distribution a certainty index of $\nu = 0.7647$ can be obtained applying (27). Higher values of $h_1(x_1), h_2(x_2)$ in the parabolic certainty distribution for x_1, x_2 result in a higher certainty index. In the Fig. 6, the

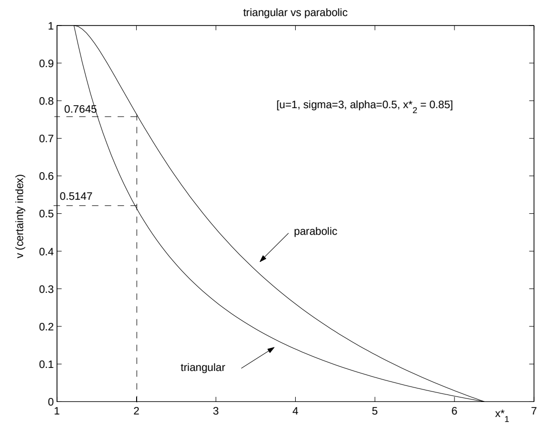

Fig. 5. A comparison between a triangular and a parabolic distribution.

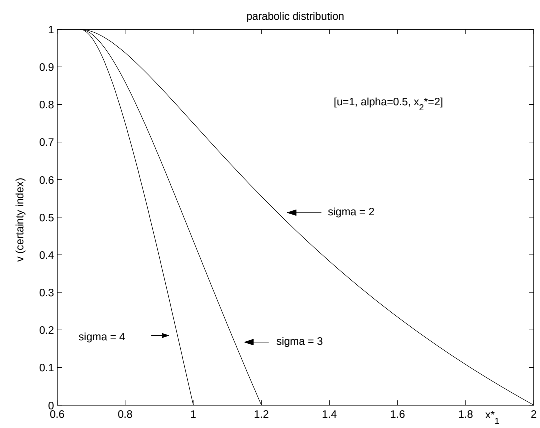

Fig. 6. Dependency on σ in a parabolic distribution.

relationship between the certainty index ν and σ is shown. For fixed values of $u = 1$, $\alpha = 0.5$, $x_2^* = 2$, the certainty index strongly depends on the value of σ.

The influence of the certainty distribution on the certainty index can be seen with the next example. For $u = 8$, $\sigma = 12$, $x_1^* = 3.2$, $x_2^* = 3$, $\sigma = 10$, the certainty index for a triangular distribution calculated by (26) is $\nu = 0.6875$. For the same values of u, σ, x_1^*, x_2^*, and $\sigma = 4$, the certainty index $\nu = 0.8750$. On the other hand, the fact that the index ν decreases when σ is being increased implies that the certainty of success of the vehicle model definition is higher when the problem constraints are less restrictive.

V. Conclusions

The uncertain variables are proved to be a useful way of studying complex problems in the field of the intelligent transport systems. In the paper presented, a real complex data association problem has been studied according to its parameters and different expert estimations. Selected simulations of numerical experiments have been presented in order to identify how the problem variables affect the certainty index of the solution and how the expert characterization of the certainty distribution affects the problem solution. Explanations of these simulations have been discussed. Future researches should be focused on the further development of the study proposed in this paper.

VI. Acknowledgements

The authors would like to thank the Spanish Ministerio de Fomento, European Space Agency (ESA) and the C. A. Región de Murcia for sponsoring the research activities under the grants FOM/3595/2003, GIROADS 332599 and ISIS/2I04SU009, respectively.

References

[1] J. Wang and C. Wilson, *Safety at the Wheel. Improving KGPS/INS Performance and Reliability*. GPS World pp. 16–26 May 2003.

[2] D. Berdjag and D. Pomorski, *DGPS/INS data fusion for land navigation*. Fusion 2004. Stockholm, Sweden, June 2004.

[3] C. Boucher and A. Lahrech and J.C. Noyer, *Non-linear filtering for land vehicle navigation with GPS outage*. IEEE Int. Conference on Systems, Man and Cybernetics. The Hague, The Netherlands, October 2004.

[4] A. Lahrech and C. Boucher and J.C. Noyer, *Fusion of GPS and odometer measurements for map-based vehicle navigation*. IEEE Int. Conference on Industrial Technology. Hammamet, Tunisia, December 2004.

[5] B. Liu and M. Adams and J. Ibaez-Guzman, *Multi-aided Inertial Navigation for Ground Vehicles in Outdoor Uneven Environments*. IEEE Int. Conference on Robotics and Automation. Barcelona, Spain, April 2005.

[6] C. Hay, *Turn, Turn, Turn. Wheel-Speed Dead Reckoning for Vehicle Navigation*. GPS World. pp. 37–42 October 2005.

[7] S. Rezaei and R. Sengupta, *Kalman Filter Based Integration of DGPS and Vehicle Sensors for Localization*. IEEE Int. Conference on Mechatronics and Automation. Ontario, Canada, August 2005.

[8] P.A. Boysen, H. Zunker, *Integryty Hits the Road, Low Cost, High Trust for Mobile Units*. GPS World, pp. 30–36, July, 2005.

[9] G. Chen, D.A. Grejner-Brzezinska, *Land-Vehicle Navigation using Multiple Model Carrier Phase DGPS/INS*. Proceedings of the American Control Conference, pp. 2327–2332, Arlington, USA, 2001.

[10] R. Toledo, *A High Integrity Navigation System for Vehicles in Unfriendly Environments*. PhD. Thesis. University of Murcia. Murcia, Spain. November 2005.

[11] Y. Bar-Shalom, W. Dale Blair, *Multitarget-Multisensor Tracking: Applications and Advances*.Vol III. Artech House, 2000.

[12] D. Huang, H. Leung, *EM-IMM Based Land-Vehicle Navigation with GPS/INS*. Proceedings of the IEEE ITSC Conference, pp. 624–629, Washington, USA. October 2004.

[13] Abbas, A.J et al. *Public transportation schedule based on modular controlled stochastic Petri nets*. Proc. of 15th Triennial World Congress of IFAC. Barcelona, 2002.

[14] Figueiredo, L. et al. *Research issues in intelligent transportation systems* Proc. of the European Control Conference ECC 01. Porto, Portugal. 2001.

[15] Bubnicki, Z. *Uncertain variables and their applications for a class of uncertain systems* International Journal of Systems Science, 32, pp. 651–659. 2001.

[16] Bubnicki, Z. *Uncertain variables and their application to decision making* IEEE Trans. On SMC, Part A: Systems and Humans, 31, pp. 587–596. 2002.

[17] Bubnicki, Z. *Application of uncertain variables to decision making in a class of distributed computer systems* Proc. of 17th IFIP World Computer Congress 2002.

[18] Bubnicki, Z. *Stability and stabilization of discrete systems with random and uncertain parameters* Proc. of 15th Triennial World Congress of IFAC, vol. E (Camacho E. F., Basanez L. and de la Puente J. A.) pp. 193-198. Pergamon, Oxford, 2002.

[19] Bubnicki, Z. *Analysis and Decision Making in Uncertain Systems* Springer-Verlag, Berlin, London, N.York. 2004.

[20] B. Barshan and H. F. Durrant-Whyte, *Inertial Navigation Systems for Mobile Robots*. IEEE Internatinal Transactions on Robotics and Automation. Vol. II NO. 3: 328–342. June 1995.