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Hybridized GPS/DR Positioning System With Unknown Initial Heading For Land Vehicles

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Abstract— **In the Intelligent Transportation Systems (ITS) field, the number of applications that demand a high integrity positioning system is growing. In order to improve the integrity of localization systems, GPS is usually hybridized with additional proprioceptive sensors. In this paper, a new hybridization algorithm based on GPS plus odometry and a gyro is proposed as an improvement of the most common extended Kalman filter (EKF) approach. In concrete, these investigations focus on the performance of the system under bad initial conditions. Results show the suitability of the proposed system for navigation under bad initial values of heading, and its benefits as compared to two state-of-the-art methods of the literature: an EKF, and a particle filter based solution.**

I. INTRODUCTION

New location based services and advanced driver assistance systems (ADAS) provide new interesting features in our road vehicles. Most of these applications need a high integrity localization system, and low cost OBEs (On Board Equipments). However, solutions based exclusively on a single GNSS receiver cannot guarantee reliable positioning in unfriendly environments [1].

Main requirements regarding localization systems concern continuity of an accurate positioning, fault detection and the provision of an integrity parameter [1]. These features allow the creation of more complex ADAS applications. For example, in some collision avoidance support systems (CASS), a decision making process encourages the creation and interpretation of a scene in order to determine the vehicle role in its environment [2].

In mass market location based applications, cost considerations must be taken into account. In the case of navigation systems, these considerations concern sensor configurations and technologies, and the complexity of the computational system.

In previous works of our group, different sensor configurations, vehicle models and fusion methods were analyzed [3], [4], [5]. In this occasion, our investigations are centered in the problem of bad initial conditions in the performance of an hybridized GPS/DR-based navigation systems.

In our research, we have adopted a sensor configuration consisting of GPS plus odometer and a gyro to measure the yaw angular rate. This configuration avoids the lack of precision in the heading of the simplest hybrid system based on GPS plus odometry (ABS system or odometer), and it is less complex than some GPS/INS hybrid localization systems [1]. This hybrid system is able to provide positioning in GPS outage situations.

The commented configuration provides reliable positioning most of the time but it presents some lacks in case that high integrity positioning is required. In a real-time system, initial steps after a reset cause a gap where the heading of the vehicle needs to be known to start the estimation (integration of the yaw angular rate). Most of the authors extract the heading from the first GPS points. This is the simplest way, but some considerations should be taken into account, for instance if the vehicle is turning. There is a great interest in the resolution of this problem in the last years [7], [8]. One interesting option to improve initial estimation in a real time system is the particle filter (PF) [9], [10]. With a PF, information about the initial conditions is not necessary, at the expense of much higher computational costs (dependent on the number of particles) as compared to some other methods [5]. In this paper, we present a new method based on GPS points, suitable for initial heading estimation in curves. Performances of the proposed method, the most commonly adopted EKF approach, and a particle filter are compared in the paper, proving the suitability of our proposal.

II. ALGORITHM DESIGN

This section details the kinematic model and the different data fusion tools for position estimation used in the performed tests. The sensor fusion method is based on the Extended Kalman Filter with special attention to the initial conditions.

A. Vehicle model with EKF

An important aspect of the positioning is the employed vehicle model. On one hand, one-dimensional models used in rail carriages are discarded in road applications. On the other hand, three-dimensional GPS/INS systems [11] usually used in aircraft are sometimes used in road vehicles too, bringing the possibility to use strap-down inertial measurement units that can offer valuable information on all the degrees of freedom of the vehicle. But three-dimensional models imply complexity and high computational cost. Kinematic two-dimensional models are the most popular option [12], providing the evolution of the vehicle pose in a two-dimensional reference frame, in a compliant way with the non-holonomic constraints.

Due to the road profile in these experiments (without lane changes and abrupt maneuvers), we consider unnecessary in

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this case the use of multi-model based approaches [1], [2], and others dynamic considerations about the vehicle.

In this research, we use the classical 2D kinematic model of a vehicle in a plane (position of a point of the vehicle plus yaw angle), that is shown in discrete time with the following equation:

$$\begin{aligned} x(k+1) &= x(k) + ds(k)cos(\psi(k) + w(k)T/2) \\ &\quad -w(k)T(D_x sin\psi(k) + D_y cos\psi(k)) \\ y(k+1) &= y(k) + ds(k)sin(\psi(k) + w(k)T/2) \\ &\quad +w(k)T(D_x cos\psi(k) - D_y sin\psi(k)) \\ \psi(k+1) &= \psi(k) + w(k)T \end{aligned} \quad (1)$$

where $(x(k), y(k))$ is the NE (North-East) projected position of the center of the GPS antenna in the vehicle, $\psi(k)$ the yaw angle, and T the sampling period. The increment of travelled distance $ds(k)$ and angular rate $w(k)$ are measured by the odometer and the gyroscope respectively. The odometer is assumed to be situated at the rear wheels axle, and the gyroscope is fixed in the vehicle body frame. The x-axis and y-axis correspond to the North and the East respectively. (D_x, D_y) are the coordinates of the GPS antenna in the body frame.

In the classical 2D model it is assumed that the trajectory is locally linear or circular. Exact equations in case of circular trajectory would exhibit $sinc$ (sinus cardinal) of the angular increment, which we can approximate to unity due to their small value.

B. Modified EKF with yaw observation from GPS data

The previous model representing the pose of the vehicle may be improved by taking into account an important aspect, the gyro bias. This bias may be constant or may change with time, being temperature-dependent. If the algorithm is left in open loop, without GPS updates, this bias increases quickly, resulting in a deviation of the path from the true one. As an example, in one of the tests, a bias of $3.910^{-4} rad/s$. was found, what caused the yaw angle to drift about $24.6°$ during the 18 minutes of the experiment. Fortunately, this bias can be estimated, as it is a systematic error, and this will be done in this section. The bias should not be mistaken for *random walk*, which appears due to the integration of noise.

One good method to improve the system performance in this aspect is the estimation of the bias by adding a fourth state variable (ξ). The gyroscope bias is estimated then through the Kalman gain. In order to let the Kalman filter estimate the bias, the covariance matrix of the state needs to be updated properly at every prediction step. Thus, the two Jacobian matrices have to be recomputed.

Now, the equations for updating the state variables become:

$$\begin{aligned} x_{k+1} &= x_k + ds_k \cos\left(\psi_k + \tfrac{(\omega_k - \xi)\,T}{2}\right) \\ &\quad -(\omega_k - \xi)T(D_x \sin\psi_k + D_y \cos\psi_k) \\ y_{k+1} &= y_k + ds_k \sin\left(\psi_k + \tfrac{(\omega_k - \xi)\,T}{2}\right) \\ &\quad +(\omega_k - \xi)T(D_x \cos\psi_k - D_y \sin\psi_k) \\ \psi_{k+1} &= \psi_k + (\omega_k - \xi)\,T \\ \xi_{k+1} &= \xi k \end{aligned} \quad (2)$$

where Jacobians respect to the state F^s and the input F^u can be simplified taking into account following assumptions:

$$\begin{aligned} \omega_k - \xi_k &\approx \omega_k \\ \sin\left(\psi_k + \frac{\omega\,T}{2}\right) &\approx \sin\psi_k \\ \sin\left(\psi_k + \frac{\omega\,T}{2}\right) &\approx \cos\psi_k \end{aligned} \quad (3)$$

As the P matrix increases, an initialization for its last row and column should be provided. This reflects our knowledge about the bias. The bias is not correlated with the vehicle position or orientation. On the contrary, it depends on the temperature and electrical noise. Therefore, the non-diagonal elements of P on the fourth row and column should be 0. The remaining element, p_{44}, should reflect the probability distribution of the bias for the family of gyroscopes used. A higher grade gyroscope will usually have lower biases. For the gyro used in our experiments, the offset was assumed to be between $-2 \cdot 10^{-3}$ and $2 \cdot 10^{-3}$ rad/s in 95% of situations. This interval is the 2σ confidence interval for a normal Gaussian distribution, and the variance is equal to $\lambda^{\xi} = (\sigma^{\xi})^2 = 10^{-6}(rad/s)^2$.

According to our experiments, both filters, the original EKF and the EKF-bias, still have troubles converging to the real yaw angle for some particular values when the initialization provided is wrong, especially when the difference between the initial angle provided and the real angle is close to $180°$ and the GPS data is corrupted by noise.

In order to overcome this limitation, another modification of the EKF algorithm is proposed. The absolute yaw angle will be estimated for every GPS reading, as the angle between the current and some previous GPS points.

Considering Fig. 1, let us assume that the yaw should be estimated using two GPS measurements, these being the latest GPS reading, at time step k, and one past GPS reading at time step $k-p$. The vehicle orientation at time step $k-p$ does not have to be known at all.

If the two GPS measurements were taken one after another, one may assume that the vehicle did not change its orientation between them, and therefore, the yaw estimation may be done by computing the absolute orientation between the two GPS points, using the $atan2$ (arc-tangent) function. In our case, we assume that the vehicle has changed its orientation between the two GPS measurements, and the amount of yaw change will be estimated from odometry data.

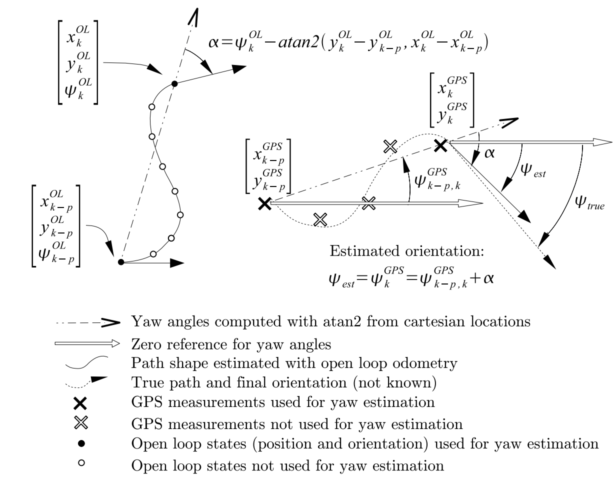

Fig. 1. Yaw estimation from GPS and odometry measurements.

The odometry data used here are not corrected in the Kalman update step. Therefore, it is necessary to run a separate processing thread that computes the odometry data in open-loop mode. Variables computed in the open loop mode will be referred as x_k^{OL}, y_k^{OL}, x_{k-p}^{OL}, etc. GPS readings at time step k will be x_k^{GPS} and y_k^{GPS}. Note that, in normal driving conditions, the odometry sampling frequency would be higher, and there will not be GPS measurements available at every instant k. For yaw estimation, it is only needed to have GPS readings at time steps k and $k-p$.

In Fig. 1, in the open loop odometry estimation of the path (left), the initial angle is unknown and can be initialized to 0 or any other value, since this is not used in computations.

By measuring the orientation $\psi_{k-p,k}$ of the straight line that connects the car position at time step $k-p$ and time step k, it is possible to estimate the current vehicle orientation ψ_k. The difference between ψ_k and $\psi_{k-p,k}$ can be computed from the open-loop estimation of the path. Since the initial angle is unknown, the path estimated by open-loop odometry will have the same shape as the true path with different orientations. Let $\psi_{k-p,k}^{OL}$ be the orientation of the line connecting $\left(x_{k-p}^{OL}, y_{k-p}^{OL}\right)$ and $\left(x_k^{OL}, y_k^{OL}\right)$, and let α be the difference between $\psi_{k-p,k}^{OL}$ and ψ_k^{OL}. If all the measurements were ideal, the difference α would be equal to the difference between $\psi_{k-p,k}$ and the real vehicle orientation at time step k.

Therefore, one may use the two GPS measurements to estimate $\psi_{k-p,k}$ and correct it with α to estimate ψ_k:

$$\psi_{k-p,k}^{GPS} = \operatorname{atan2}\left(y_k^{GPS} - y_{k-p}^{GPS},\ x_k^{GPS} - x_{k-p}^{GPS}\right) \quad (4)$$

$$\alpha = \psi_k^{OL} - \operatorname{atan2}\left(y_k^{OL} - y_{k-p}^{OL},\ x_k^{OL} - x_{k-p}^{OL}\right) \quad (5)$$

$$\psi_k^{GPS} = \psi_{k-p,k}^{GPS} + \alpha \quad (6)$$

Equations (4)-(6) show a computationally-inexpensive way to estimate the yaw angle at every time step k, provided that there is available a past GPS measurement at time step $k-p$. The complexity of the algorithm increased mainly by adding a second state vector $(x_k^{OL}, y_k^{OL}, \psi_k^{OL})$ that has to be updated every time a new odometry measurement is available, and by the two uses of the $atan2$ function in yaw estimation.

To integrate the yaw estimation into the Kalman loop, it is necessary to compute its variance. The variance of the yaw estimation is the sum of three terms:

$$\lambda_k^{\psi} = \lambda_k^{\psi,OL} + \lambda_{k-p,k}^{\psi,GPS} + \lambda_{k-p,k}^{\psi,OL} \quad (7)$$

The first term appears due to the integration of the noise from the gyroscope readings. Since the variance $\lambda^{\omega} = (\sigma^{\omega})^2$ is considered constant, this term can be computed using a non-recursive equation:

$$\lambda_k^{\psi,OL} = k\left(\frac{\sigma^{\omega}}{T}\right)^2 \quad (8)$$

In order to compute the variance from the two GPS readings, a simplification hypothesis will be used. Let us suppose, without loss of generality, that at step $k-p$ the vehicle was at $x=0$, $y=0$, and at step k it was at $x=d$, $y=0$. These are the ideal locations, uncorrupted by noise. The orientation of the line connecting these two location is 0. The noise from the GPS receiver is considered to be Gaussian, with the following model:

$$\begin{aligned} x_{k-p}^{GPS} &= 0 + \varepsilon_{k-p}^{x}, \quad \varepsilon_{k-1}^{x} = \mathcal{N}(0, \lambda^{GPS}) \\ y_{k-p}^{GPS} &= 0 + \varepsilon_{k-p}^{y}, \quad \varepsilon_{k-1}^{y} = \mathcal{N}(0, \lambda^{GPS}) \\ x_k^{GPS} &= d + \varepsilon_k^{x}, \quad \varepsilon_k^{x} = \mathcal{N}(0, \lambda^{GPS}) \\ y_k^{GPS} &= 0 + \varepsilon_k^{y}, \quad \varepsilon_k^{y} = \mathcal{N}(0, \lambda^{GPS}) \end{aligned} \quad (9)$$

It is also assumed that there is no correlation between the noise for X and Y axes, therefore the covariance matrix of GPS measurement noise is diagonal, and the uncertainty ellipse is a circle:

$$R_k = \begin{bmatrix} \lambda^{GPS} & 0 \\ 0 & \lambda^{GPS} \end{bmatrix} \quad (10)$$

Let $r_{xx}(i) = E\left\{\varepsilon_k^x \, \varepsilon_{k-i}^x\right\}$ be the autocorrelation of the X measurement noise from GPS, and $r_{yy}(i)$ its counterpart for Y axis. Obviously, $r_{xx}(0)$ and $r_{yy}(0)$ will be equal to λ^{GPS}. Since the noise from the two axes is considered uncorrelated, $r_{xy}(i)$ will be 0 $\forall i$.

For variance analysis, $\psi_{k-p,k}^{GPS}$ will be linearized around 0.

$$\psi_k = 0 + \varepsilon_k^{\psi} \quad (11)$$

Here, ε_k^{ψ} will not be Gaussian, but for large values of d, its distribution will be approximated to a normal Gaussian.

The angle measurement function will be also linearized around d. These two approximations will be realistic as much as the distance d is much greater than the standard deviation of the GPS measurements, σ^{GPS}.

$$\begin{aligned} \psi_k^{GPS} &\approx \frac{y_k^{GPS} - y_{k-p}^{GPS}}{x_k^{GPS} - x_{k-p}^{GPS}} = \frac{\varepsilon_k^y - \varepsilon_{k-p}^y}{d + \varepsilon_k^x - \varepsilon_{k-p}^x} \\ &\approx \frac{1}{d}(\varepsilon_k^y - \varepsilon_{k-p}^y) \end{aligned} \quad (12)$$

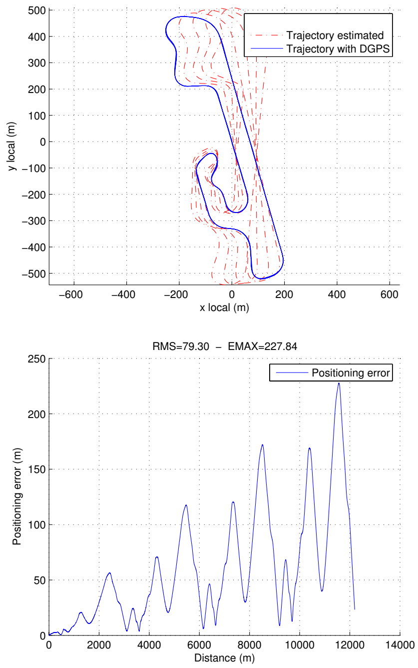

Fig. 2. Open loop test (no GPS measurement) without gyro bias compensation.The test was performed during four laps around the shown circuit.

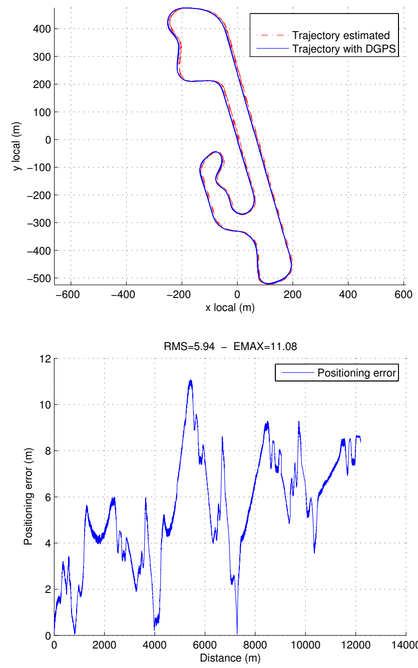

Fig. 3. The open loop test with bias compensation.

Computing the auto-covariance of $\varepsilon^{\psi} = \psi^{GPS}$ for large values of d with respect to σ^{GPS}, by taking into account that $r_{xy}(i) = 0 \ \forall i$:

$$\begin{aligned} E\left\{(\varepsilon^{\psi})^2\right\} &= E\left\{\frac{1}{d^2}\left(\varepsilon^y_k - \varepsilon^y_{k-p}\right)^2\right\} \\ &= E\left\{\frac{1}{d^2}\left[(\varepsilon^y_k)^2 + (\varepsilon^y_{k-p})^2 - 2\,\varepsilon^y_k\,\varepsilon^y_{k-p}\right]\right\} \\ &= \frac{2}{d^2}\left[r_{yy}(0) - r_{yy}(p)\right] \end{aligned} \quad (13)$$

Since the errors for X and Y axes are considered to have the same autocorrelation sequence, equal to $r^{GPS}(i)$, the variance of the yaw measurement error is:

$$\lambda^{\psi,GPS}_{k-p,k} = E\left\{(\psi^{GPS})^2\right\} = \frac{2}{d^2}\left[\lambda^{GPS} - r^{GPS}(p)\right] \quad (14)$$

C. Particle Filter

An alternative to solve the problem of unknown initial yaw is the use of a particle filter. The basic idea of the particle filter is to represent by means of a set of N samples $\{X^i_k\}^N_{i=1}$ and their corresponding weights $\{w^i_k\}^N_{i=1}$ the probability density function $p(X_k/Y_{1:k})$ at instant k of the state vector $\mathbf{X}_k$, given past observations, following next numerical expression:

$$p(\mathbf{X}_k/Y_{1:k}) \simeq \sum_{i=1}^{N} X^i_k \cdot w^i_k$$

where $Y_{1:k}$ stands for the observations collected from the initialization till instant k. The process of particle filtering designed for our algorithm can be summarized as follows:

1) Initialization: Generation of N particles, or samples of the state vector, X^i_0, with equal weights $1/N$.
2) Prediction: Estimation of X^i_{k+1} following the model dynamics.
3) Measurement update: Update of the weights of the particles with the observations Y_k, following the expression

$$w^i_k = w^i_{k-1} \cdot e^{\left(-0.5(Y_k - h(X^i_k))'(R^{-1})(Y_k - h(X^i_k))\right)}$$

where $h(X^i_k)$ is the observation function that relates at instant k the state X^i_k and observations Y_k (in our case it will be the second order identity matrix), and R the covariance matrix of observations.
4) Normalization of the weights: $w^i_k = w^i_k / \sum_{i=1}^{N} w^i_k$.
5) Resampling: To prevent high concentration of probability mass at only a few particles, (leading to the convergence of a single w^i_k to 1), particles are resampled when next inequation is verified

$$\frac{1}{\sum_{i=1}^{N}(w^i_k)^2} < 0.5N$$

6) End of cycle: Making $k = k + 1$, and iterating to step 2.

Further details of particle filtering can be found in [13].

III. EXPERIMENTAL TRIALS

A. Test conditions and onboard equipment

To test the algorithms, we employed real data-sets lent for this purpose by the Division of Metrology and Instrumentation of LCPC Nantes Centre, and collected during a three kilometers long circuit near Nantes, west of France

[14]. These experiments were carried out with a Peugeot van equipped with a wheels axle odometer (the a priori calibrated spatial rate is 1 pulse per 24.15 cm), a fiber optic gyroscope KVH 2100 e-core FOG series, a Crowsbow IMU VG-400 (MEMS technology), and a DGPS receiver Trimble Ag132 receiving Omnistar corrections. The maximum speed during the tests was 100 km/h and its average value 50 km/h.

These circuits offer good conditions for GPS observation. Therefore, we artificially simulated GPS masks. A 400 seconds GPS mask was applied after another 400 seconds period with GPS signal. During the rest of the test, GPS signals were enabled again. The GPS noise has been modeled as gaussian with $\sigma = 1m$ in all the performed experiments.

B. Results

First of all, the influence of the gyro bias in the performance of the localization system will be tested. Since all the tests performed have long gaps without GPS readings, it is highly recommended to take into account the gyro bias (in other case the heading of the vehicle degrades the estimation). In Figs. 2 and 3, it can be seen the influence of the gyro bias compensation.

Fig. 4 shows the performance of three algorithms, the EKF, EKF-bias and the EKF-bias-yaw in the same circuit shown in Figs. 2 and 3, with a long GPS mask (400 s). During the mask, the influence of the bias estimation is considerable. The lowest image of the figure shows the performance of the initial yaw estimation algorithm. The RMS error and maximum error are slightly lower than the EKF-bias values due of the faster convergence during the initial steps.

We have seen that the bias compensation is highly advisable in this kind of experiment, more than the model of the vehicle itself. Nevertheless, the main purpose of this research is the study of the performance of the localization system with bad initial heading value.

Previously in [5], an approach with particle filter was tested. A particle filter may allow to converge faster to the real initial heading, but its main problem is that a high number of particles increases the computational cost. In this work, it is presented the computation of the initial conditions necessary for the traditional fusion tools based on Kalman filter with a very simple algorithm, and taking into account GPS measurements. The algorithm is able to find the initial heading even if the vehicle is turning during the estimation of the heading through GPS measurements. Both the particle filter and the EKF-bias-yaw proposed have been compared with the standard EKF.

The standard EKF has noticeable difficulties in converging to the real yaw angle when the initial value provided was far from the real one (eg. $real\_yaw + \pi$ radians). As a matter of fact, the filter "thought" that the vehicle was oriented in the opposite direction. As the yaw angle was not an observable variable, EKF tried to correct it through linearization, which did not work properly.

In the test shown in Figs. 5 and 6, the yaw angle was initialized to $real\_yaw + \pi$. The 400-second mask from the previous test was also applied here, to evaluate the ability of

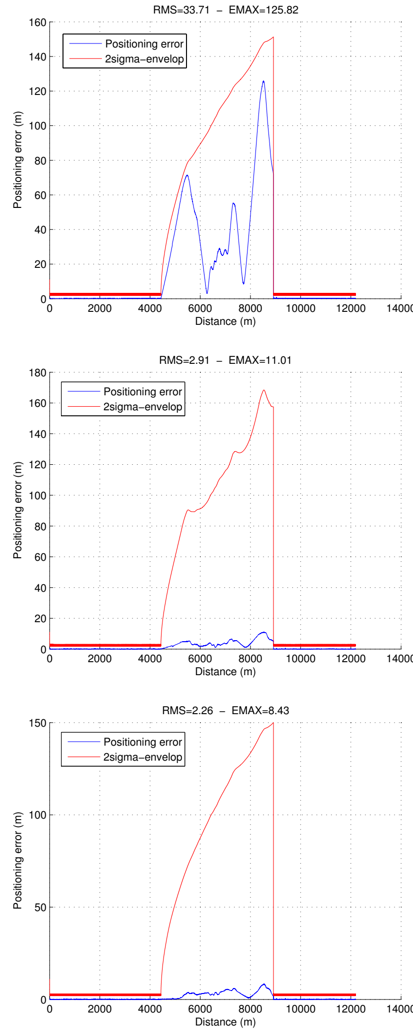

Fig. 4. Performance of the algorithms during a long GPS mask (400 s). Upper: EKF, middle: EKF-bias, lower:EKF-bias-yaw.

the EKF-bias-yaw and the PF to estimate the position even when a wrong initial angle was provided.

Fig. 5 shows how the DGPS reference and the EKF-bias-yaw and PF estimations appear very similar. Fig. 6 shows how the EKF algorithm needs more than 800 m to converge, and the EKF-bias-yaw and the PF (implemented with 500 particles) both show similar performance and they have the same convergence speed (both were able to converge during first 50 m, or about 70 iterations). The performance of the positioning error with the EKF-bias-yaw is even slightly better than the PF. EKF-bias-yaw was also able to estimate the sensor bias and it performs the open-loop tests with only slightly lower performance than in the test with good yaw angle initialization.

IV. CONCLUSIONS

This paper presents an hybrid localization system based on the fusion of GPS, odometry and a gyro, capable to provide high quality positioning in unfriendly conditions.

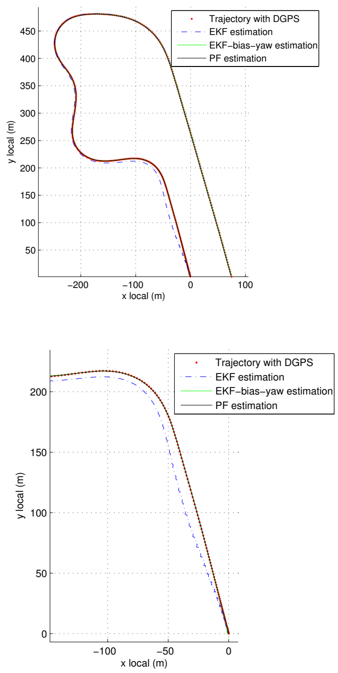

Fig. 5. Estimated and reference trajectories corresponding to the first part of the circuit, and a zoom of the initial steps.

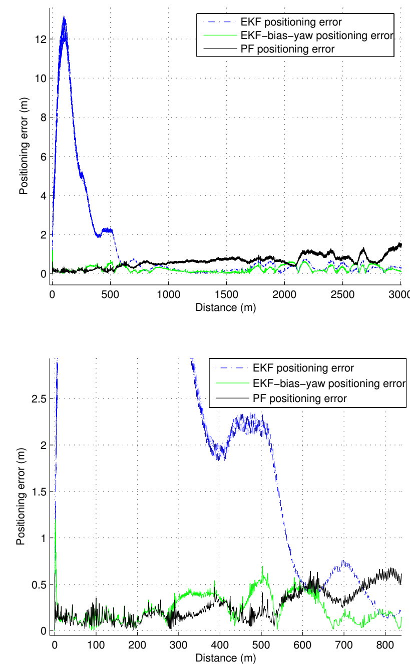

Fig. 6. Positioning error with EKF, EKF-bias-yaw and the PF during to the first 3000 m. of the circuit, and a zoom of the initial steps.

On the one hand, the filter overcomes the negative influence of the gyro bias in the localization system. A real time bias estimation has been proposed in this respect. On the other hand, the problem of bad or unknown initial conditions has been particularly studied. A novel algorithm for initial heading estimation that can be applied to Kalman filter based solutions was developed and tested. Experimental trials with real data-sets show the very good results obtained by the proposed method.

These results are comparable to those achieved by a particle filter based solution that is much more costly in computational terms.

V. ACKNOWLEDGMENTS

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