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Coupled multiple time scale piecewise linear oscillators. Application to a neuroendocrine system

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Piecewise linear (PWL) systems are a family of non-smooth systems that reproduce faithfully the dynamics of models coming from applications. Initially, in the seminal book of Andronov et al. [1], PWL systems were developed in order to model engineering problems, but they have been also able to capture social behaviors, financial or biological problems [5]. The rigorous analysis of PWL systems has revealed that they not only exhibit as rich dynamics as smooth systems, but can also feature new behaviors that are impossible to obtain under differentiability assumptions.

The FitzHugh-Nagumo (FHN) system models, as it is well-known, the electrical activity of neurons. It corresponds to a simplified planar version of the celebrated Hodgkin-Huxley model. The main assumption underlying conductance-based neuron models is that a neuron behaves as an electronic circuit, which have been successfully modeled by PWL systems. In [2, 3, 4], a four dimensional system was constructed and analyzed modeling the pulse and surge pattern of gonadotropin releasing hormone (GnRH) secretion by hypothalamic neurons in female mammals. The model consists of two coupled FHN systems running on different time scales. One system models the average secretory activity of GnRH neurons, while the other system corresponds to the average activity of regulatory neurons. The resulting model is studied both qualitatively and quantitatively. In the work that we present here [6], we have replaced the FHN subsystems by two PWL equivalent, namely McKean caricatures, where the original cubic function is replaced by a PWL function that preserves the cubic shape. This change allows us to obtain more information on the system and to compute explicitly the solutions in each linearity zone. Indeed, we obtain explicit formulas that had only an implicit counterpart in the smooth case.

Referencias

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