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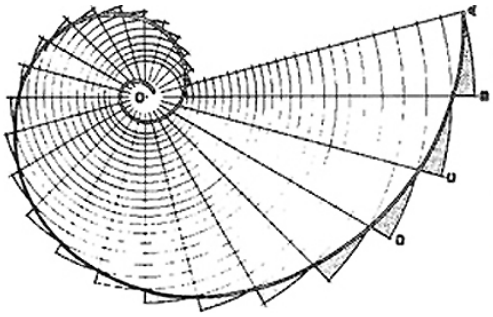
CONGRESO DE JÓVENES INVESTIGADORES

de la Real Sociedad Matemática Española

Universidad de Murcia, 7-11 Septiembre, 2015



SESIÓN

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The slicing problem, the variance conjecture, and the spectral gap conjecture

David Alonso Gutiérrez[1], **Jesús Bastero**[2]

We will present three well known open problems in the theory of convex bodies and, more generally, in the study of log-concave measures. These three problems are the slicing problem, also known as the hyperplane conjecture, the variance conjecture (which first appeared in the study of the central limit problem for convex bodies), and the Kannan-Lovász-Simonovits spectral gap conjecture. We will explain the relation between them and will show our recent results in the study of a generalized variance conjecture for hyperplane projections of the ℓ_p^n-balls.

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The maximum relative diameter for multi-rotationally symmetric planar convex bodies

Antonio Cañete[1]

In this talk we shall study the maximum relative diameter functional in the class of multi-rotationally symmetric planar convex bodies. A set C of this class is, by definition, k-rotationally symmetric for $k \in \{k_1, \ldots, k_n\} \subset \mathbb{N}$, and so it is natural dividing C into k connected subsets, with $k \in \{k_1, \ldots, k_n\}$, by using the corresponding standard k_i-partition (which is minimizing for the maximum relative diameter when $k \geq 3$, see [1]). We shall compare the different values of the maximum relative diameter for these standard partitions, obtaining the existing general relation and showing when all these values coincide.

Referencias

[1] A. Cañete, U. Schnell, S. Segura: Subdivisions of rotationally symmetric planar convex bodies minimizing the maximum relative diameter, preprint 2015.

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On the connection between the asymmetry of Minkowski, the completion and the Jung constant

Bernardo González Merino[1], **René Brandenberg**[1]

In this talk we tackle the relationship between three classic concepts of Convex Geometry: the asymmetry of Minkowski, the inequality of Jung, and the completeness. To do so we will make emphasis in the use of the asymmetry of Minkowski to strengthen and smoothen different geometric inequalities, as in [2].

This relationship can be summed up in a single equation, which we will motivate and prove in detail (see [1]). Finally we will show several consequences derived from this result, showing also some connections with other well known geometric measurements, such as the Banach-Mazur distance or the Helly dimension.

Referencias

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[2] R. Brandenberg, S. König, Sharpening well known geometric inequalities using computable symmetry coefficients, *Mathematika* 2014, doi:10.1112/S0025579314000291.

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On dual Minkowski-inequalities via covering minima

Matthias Henze[1], **Bernardo González Merino**[2]

The subject of the Geometry of Numbers was started by the seminal and foundational work of Minkowski in the late 19th century. Based on natural geometric ideas he proved beautiful criteria for 0-symmetric convex bodies to contain non-trivial integral points, and he applied his findings to solve number theoretic questions. This theory developed into an independent branch of mathematics that found applications also in integer programming, functional analysis, additive combinatorics, and many more.

Up until today, besides the efforts of many eminent mathematicians there is no satisfying dual theory to Minkowski's original theorems. We pick up on the duality between packing and covering arrangements, and in particular we study the so-called covering minima of convex bodies, which were introduced by Kannan & Lovász (1988) to study flatness-theorems for integer programming and diophantine approximation.

Generalizing a problem posed by Makai Jr., we formulate precise conjectures on dual Minkowski-inequalities relating the volume of a convex body to its covering minima. We solve the conjectures for the important class of unconditional convex bodies and we further explore asymptotic estimates for the general case. The occuring extremal examples give rise to an interesting family of convex bodies that may be thought of as analogs to the well-studied parallelohedra, that is, convex bodies tiling space by translation.

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Is there a p-version of the Minkowski difference? A survey

Antonio R. Martínez Fernández[1]

In this talk we introduce a notion of p-difference of convex bodies as a substraction counterpart of the well-known p-sum, for $p \geq 1$. We show several properties of this p-difference, and define the corresponding notions of p-(inner) parallel bodies and p-kernel. We prove an analog of the concavity of the family of classical parallel bodies for the p-parallel ones, as well as the continuity of this new family, in its definition parameter. Further results on inner parallel bodies are extended to p-inner ones. For instance, we show that tangential bodies are characterized as the only convex bodies all whose p-inner parallel bodies are homothetic copies of them.

Quermassintegrals are functionals associated to a convex body K which appear when computing the volume of the Minkowski addition of K and a ball. We investigate the differentiability of the quermassintegrals with respect to the one-parameter family of p-parallel bodies. As in the classical case, we obtain that the volume is always differentiable. Although there is no polynomial expression for the p-sum, the other quermassintegrals are differentiable for positive values of the parameter too.

This talk is based on joint works with M. A. Hernández Cifre, E. Saorín Gómez and J. Yepes Nicolás.

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Geometry of Random Polytopes and Orlicz Spaces

Joscha Prochno[1], **David Alonso-Gutiérrez**[2]

We present an approach to study the geometry of random polytopes that is based on probabilistic inequalities for random vectors in terms Orlicz norms. To be more precise, we relate a natural Orlicz function to a convex body K in $\mathbb{R}^n$. This Orlicz function is then used to study geometric functionals on random polytopes generated by an isotropic K.

Referencias

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The isoperimetric problem inside a Euclidean convex body

César Rosales[1]

In a convex body Ω of $\mathbb{R}^n$ we consider the isoperimetric problem of minimizing the perimeter under a volume constraint. In our context the perimeter measures the whole boundary area, including the contribution of $\partial\Omega$. By the classical isoperimetric inequality the minimizers are all round balls provided the enclosed volume is less than or equal to the volume of a largest ball in Ω. Though it is difficult to characterize minimizers of greater volumes, it seems reasonable to expect that they are always convex. This is an open question posed by Stredulinsky and Ziemer in 1997. In this talk we will review some old and new results in relation to this conjecture.

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Classification of Curvature Measures with Values in Irreducible SO(n)-Representations

Mykhailo Saienko[1]

Hug, Schneider and Schuster have classified all valuations and curvature measures with values in symmetric tensors (which they call local tensor valuations) in Euclidean spaces. We generalize several their results by giving a classification of all curvature measures and valuations with values in arbitrary SO(n)-representation together with differential forms inducing them. The discovered differential forms may provide an efficient means of studying the module structure and the (semi-local) kinematic formulae for this extended class of curvature measures.

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Contact measures in isotropic spaces

Gil Solanes[1]

Contact measures were introduced by Firey, and were subsequently studied by Schneider and Teufel among others, to measure the set of positions in which a moving body touches a fixed one. After revisiting this subject under the new perspective of the theory of valuations on manifolds, we will show how contact measures provide a link between the integral geometry of curved isotropic spaces and the kinematic formulas for area measures recently studied by Wannerer.

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On Brunn-Minkowski type inequalities for polar bodies

Jesús Yepes Nicolás[1], **María A. Hernández Cifre**[2]

In this talk we will discuss about some Brunn-Minkowski type inequalities involving the notion of polarity of convex bodies. More precisely, we will show a generalization for the p-sum of convex bodies of previous results by Firey for the polar set, as well as an equivalent multiplicative version of it, which also provides an alternative proof for Firey's results.

On the other hand, we will make some considerations for the polar set of the so-called difference body with respect to the Minkowski addition and p-sum, and we will discuss about the more convenient operation that should be taken into account in order to get a Rogers-Shephard type inequality.

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