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On dual Minkowski-inequalities via covering minima

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The subject of the Geometry of Numbers was started by the seminal and foundational work of Minkowski in the late 19th century. Based on natural geometric ideas he proved beautiful criteria for 0-symmetric convex bodies to contain non-trivial integral points, and he applied his findings to solve number theoretic questions. This theory developed into an independent branch of mathematics that found applications also in integer programming, functional analysis, additive combinatorics, and many more.

Up until today, besides the efforts of many eminent mathematicians there is no satisfying dual theory to Minkowski's original theorems. We pick up on the duality between packing and covering arrangements, and in particular we study the so-called covering minima of convex bodies, which were introduced by Kannan & Lovász (1988) to study flatness-theorems for integer programming and diophantine approximation.

Generalizing a problem posed by Makai Jr., we formulate precise conjectures on dual Minkowski-inequalities relating the volume of a convex body to its covering minima. We solve the conjectures for the important class of unconditional convex bodies and we further explore asymptotic estimates for the general case. The occuring extremal examples give rise to an interesting family of convex bodies that may be thought of as analogs to the well-studied parallelohedra, that is, convex bodies tiling space by translation.

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