Einstein G_2 manifolds obtained as warped products

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The presence of a G_2-structure on a manifold M is equivalent to the existence of a certain 3-form φ on M. Different classes of special G_2-structures can be described by the behavior of the 3-form φ. For example, a G_2-structure is called *calibrated* if φ is closed, and *cocalibrated* if φ is coclosed, that is, if $*\varphi$ is closed with $*$ denoting the Hodge star operator. In the latter case, if $*\varphi$ is proportional to $d\varphi$, then the G_2-structure φ is said to be *nearly parallel*.

As it was shown in [2] the behavior of the Ricci tensor associated to the metric g_φ is closely related with the behavior of the G_2-structure φ. By the results in [3] no compact 7-dimensional manifold can support a calibrated G_2-structure φ whose underlying metric g_φ is Einstein unless g_φ has holonomy contained in G_2. However, 7-dimensional manifolds with a nearly parallel G_2-structure are always Einstein.

Using warped products, we show how to construct manifolds endowed with special G_2-structures from manifolds endowed with different classes of $SU(3)$-structures in such a way that the Einstein condition on the corresponding metric is preserved along this construction.

Referencias

[1] A. Besse: *Einstein manifolds*. Springer, Berlin, Heidelberg, New York, 1987.

[2] R. L. Bryant: Some remarks on G_2-structrures. In *Proceedings of Gökova Geom.-Topology Conference 2005*, 75–109. Gökova Geometry/Topology Conference (GGT), Gökova, 2006.

[3] R. Cleyton, S. Ivanov: On the geometry of closed G_2-structures, *Commun. Math. Phys.* 270 (2007), 53–67.

[4] B. O'Neill: *Semi-Riemannian Geometry with Applications to Relativity*. Pure and Appl. Math. **103**, Academic Press, New York, 1983.

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