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Simultaneous p-orderings and minimising volumes in number rings

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Let A be a domain and K be its field of fractions. We call a polynomial $f \in K[X]$ integer-valued if $f(A) \subseteq A$. A subset $S \subseteq A$ is called n-universal if for every polynomial of degree at most n the following condition is satisfied: $f(S) \subseteq A$ if and only if $f(A) \subseteq A$. For example, the set $\{0, \ldots n\}$ is an n-universal set in $\mathbb{Z}$. The notion of n-universal set was defined by Petrov and Volkov in [1] and is connected to the notion of p-ordering introduced by Bhargava. Petrov and Volkov in [1] studied the minimal cardinality of n-universal sets. It can be easily shown, that if the ring A is not a field an n-universal subset of A contains at least $n+1$ elements. Petrov and Volkov showed that there are no n-universal sets of size $n+1$ in $\mathbb{Z}[i]$, provided that n is large enough. In a joint work with Jakub Byszewski and Mikolaj Fraczyk we extended their result to the rings of integers in any imaginary quadratic field. Petrov and Volkov also stated a conjecture about the minimial cardinality of n-universal sets in the ring of Gaussian integers. We give a strong counterexample to their conjecture by showing that in a ring of integers of any number field, for any natural n there exists an n-universal set with only $n+2$ elements. On the way, we discover a link with Euler-Kronecker constants.

Referencias

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