

Higher genus curves and the inverse Galois problem

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Let $\overline{\mathbb{Q}}$ be an algebraic closure of \mathbb{Q} , let n be a positive integer and let ℓ a prime number. Given a curve C over \mathbb{Q} of genus g, it is possible to define a Galois representation $\rho : \operatorname{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \to \operatorname{GSp}_{2g}(\mathbb{F}_{\ell})$, where \mathbb{F}_{ℓ} is the finite field of ℓ elements and GSp_{2g} is the general symplectic group in GL_{2g} , corresponding to the action of the absolute Galois group $\operatorname{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ on the ℓ -torsion points of its Jacobian variety J(C). If ρ is surjective, then we realize $\operatorname{GSp}_{2g}(\mathbb{F}_{\ell})$ as a Galois group over \mathbb{Q} . In this talk I will describe an ongoing project with Pedro Lemos and Samir Siksek, concerning the realization of $\operatorname{GSp}_6(\mathbb{F}_{\ell})$ as a Galois group for infinitely many odd primes ℓ . The approach towards this instance of the Inverse Galois problem is based on the study of curves of higher genus, as well as combinatorial results in group theory and discriminant bounds for number fields.

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