Analytic construction of points on modular elliptic curves

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Let E be an elliptic curve over a number field F. The Birch–Swinnerton-Dyer conjecture is one of the yet unsolved Millennium problems posed by the Clay Mathematics Institute, and one of the key ingredients used to prove the known cases is the existence (when E is modular and F is totally real) of Heegner points, a systematic supply of points on E defined over certain algebraic extensions of F.

After recalling the construction of Heegner points on elliptic curves defined over $\mathbb{Q}$, I will explain how this construction was extended by Darmon and others to give constructions of points on elliptic curves via analytic methods, which were conjectured to be algebraic and satisfy properties akin to those satisfied by the Heegner points. Finally we will introduce the joint work with X.Guitart and M.H. Sengun, in which we generalize the existing construction to (modular) elliptic curves defined over arbitrary number fields.

Since we are at present unable to prove any of these conjectures, the talk will be illustrated with examples, in an attempt convince the audience of their validity.

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