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# Projection Algorithms for Convex and Nonconvex Feasibility Problems

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A *feasibility problem* requests solution to the problem

$$\text{Find } x \in \bigcap_{i=1}^{N} C_i$$

where $C_1, C_2, \ldots, C_N$ are finitely many closed sets lying in a Hilbert space $\mathcal{H}$. In this talk we consider iterative methods based on the non-expansive properties of the metric *projection* operator

$$P_C(x) := \operatorname{argmin}_{c \in C} \|x - c\|$$

or *reflection* operator $R_C(x) := 2P_C(x) - x$ on a closed convex set $C \subset \mathcal{H}$. At each step, these methods utilize the nearest point projection onto each of the individual constraint sets. The philosophy here is that it is simpler to consider each constraint separately, rather than the intersection directly. These methods are especially useful when the number of sets involved is large as the methods are fairly easy to parallelize.

Applied to closed convex sets, the behavior of projection algorithms is quite well understood. Moreover, their simplicity and ease of implementation have ensured continued popularity for successful applications in a variety of nonconvex optimization and reconstruction problems. This popularity is despite the absence of sufficient theoretical justification.

Particularly, in recent times, the Douglas–Rachford algorithm has been empirically observed to effectively solve a variety of nonconvex *feasibility problems*, including those of a combinatorial nature. In this talk we show global convergence behavior of the algorithm for solving various nonconvex feasibility problems. We also discuss recent successful applications of the method to a variety of *matrix reconstruction problems*, both convex and nonconvex. In these problems one aims to reconstruct a matrix, with known properties, from a subset of its entries.

## Referencias

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