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A look at distributions

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Consider M a smooth manifold. An m–distribution ξ on M is a smooth correspondence assigning an m–dimensional plane in T_pM to each point $p \in M$ – equivalently, it is a smooth section of $Gr_m(TM)$. Familiar examples of distributions would be foliations and contact structures. These two examples satisfy a so–called *Darboux theorem*: they all look the same locally, and hence their invariants have to be of a more global nature. Characterising the families having a Darboux theorem is an important problem in the theory of distributions.

A distinguishing feature between foliations and contact structures is their stability under local perturbations. Generic perturbations destroy the foliation structure but not the contact one. We say that in this sense contact structures are *stable*. It is an important theorem by Montgomery [2] that the only stable distributions having a Darboux theorem are the line fields, the contact structures, the even contact structures and the Engel structures. The least studied objects in that list are the Engel structures: 2–distributions that are maximally non–integrable in dimension 4. Indeed, not much was known about them until Vogel [3] proved in 2004 that every parallelizable 4–manifold admits an Engel structure.

The aim of the talk will be to discuss some general theory about distributions. If time allows, we will focus on Engel structures, introducing the main open problems in the field.

Referencias

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[2] R. Montgomery: Generic distributions and Lie algebras of vector fields, *J. Differential Equations* **103** (no. 2) (1993), 387–393.

[3] T. Vogel: Existence of Engel structures, *Ann. of Math. (2)* **169** (no. 1) (2009), 79–137.

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