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The homotopy type of the piecewise linear cobordism category

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Galatius, Madsen, Tillmann and Weiss proved in [3] that the classifying space $B\mathcal{C}_d$ of the category $\mathcal{C}_d$ of smooth d-dimensional embedded cobordisms in $\mathbb{R}^\infty$ has the weak homotopy type of a certain infinite loop space. More precisely, the main theorem of [3] states that there is a weak homotopy equivalence $B\mathcal{C}_d \simeq \Omega^{\infty-1}\mathbf{MT}\mathrm{O}(d)$, where $\mathbf{MT}\mathrm{O}(d)$ is the Madsen-Tillmann spectrum, whose space at degree N is the the Thom space $\mathrm{Th}(\gamma_{d,N}^{\perp})$ of the normal vector bundle over the Grassmannian $Gr_d(\mathbb{R}^N)$ of d-planes in $\mathbb{R}^N$. The proof of this result was later simplified by Galatius and Randal-Williams in [4] by using a *space of manifolds* $\Psi_d(\mathbb{R}^N)$ which was first introduced in [1]. There are structure maps $\Sigma\Psi_d(\mathbb{R}^N) \to \Psi_d(\mathbb{R}^{N+1})$ which make the collection of spaces $\Psi_d = \{\Psi_d(\mathbb{R}^N)\}_{N\geq 0}$ into a spectrum and the strategy of the proof given in [4] is to show that there is a zig-zag of weak equivalences

$$B\mathcal{C}_d \simeq \Omega^{\infty-1}\Psi_d \simeq \Omega^{\infty-1}\mathbf{MT}\mathrm{O}(d).$$

The purpose of this talk is to present the main result of my PhD thesis and of the forthcoming article [2], namely, that there is a weak equivalence $B\mathcal{C}_d^{PL} \simeq \Omega^{\infty-1}\Psi_d^{PL}$, where $\mathcal{C}_d^{PL}$ and Ψ_d^{PL} are respectively piecewise linear analogues of the category $\mathcal{C}_d$ of d-dimensional smooth cobordisms and the spectrum Ψ_d studied in [4]. I will begin this presentation by first giving a brief introduction to the main theorem of [3] and the proof of this result given by Galatius and Randal-Williams in [4]. In the second part of this talk, I will define the spectrum Ψ_d^{PL} and the category $\mathcal{C}_d^{PL}$ and prove that there is indeed a weak equivalence $B\mathcal{C}_d^{PL} \simeq \Omega^{\infty-1}\Psi_d^{PL}$. During this part of the talk, I will emphasize on the piecewise linear topological techniques that needed to be developed in order to translate the methods used in [4] to the piecewise linear setting. In the last segment of this presentation I will introduce a piecewise linear analogue $\mathbf{MT}\mathrm{PL}(d)$ of the Madsen-Tillmann spectrum and discuss how one might prove that there is a weak equivalence

$$\Omega^{\infty-1}\Psi_d^{PL} \simeq \Omega^{\infty-1}\mathbf{MT}\mathrm{PL}(d),$$

which is the missing step in the proof of the piecewise linear version of the result proven by Galatius, Madsen, Tillmann and Weiss.

Referencias

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