

## **Reconstructing compact metric spaces**

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It is natural to approximate topological spaces by other simpler ones. In this direction, there are many classical results (for instance [3], [4]) approximating spaces by means of inverse sequences of polyhedra. Shape Theory [1] deals with this situation approximating spaces with pathological local properties, using inverse sequences of ANR's or polyhedra. These are purely theoretical results with no possibility of computational purposes. With the advances in Computational Topology and the use of finite topological spaces in Digital Topology, there is a recent interest in the approximation and reconstruction problems from a more practical point of view.

We propose a construction for compact metric spaces, introduced in [2], to obtain some inverse sequences of different kinds of polyhedra and finite spaces. These sequences, and its inverse limits, represent the shape, homotopic and homeomorphic types of the original space. The advantage of this method is that it is constructive, so we can manage the reconstructions by hand or with a computer. We will also propose some questions concerning the stability of these approximations in spaces with additional structure.

## Referencias

- [1] S. Mardesic, J. Segal: *Shape Theory. The inverse system approach*. North-Holland Mathematical Library, 26, Amsterdam-New York, 1982.
- [2] M. Alonso-Morón, E. Cuchillo-Ibañez, A. Luzón: ε-connectedness, finite approximations, shape theory and coarse graining in hyperspaces, *Phys. D* 237 (23) (2008), 3109–3122.
- [3] P. S. Alexandroff: Untersuchungen über Gestalt und Lage abgeschlssener Mengen beliebeger Dimension, Annals of Math. 30 (1928), 101–187.
- [4] H. Freudenthal: Entwicklungen von Räumen und ihren Gruppen, Compositio Math. 4 (1937), 154–234.

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