New complex hyperbolic mixed dark soliton solutions for some nonlinear partial differential equations

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Abstract:

This work focuses on obtaining new complex hyperbolic and mixed dark solutions for some nonlinear partial differential equations, namely, (2+1)-dimensional asymmetrical Nizhnik-Novikov-Veselov and Sawada–Kotera (SK) equations via sine-Gordon expansion method. This powerful method is based on two important properties of sine-Gordon equation. We generate new solitary wave solutions to the governing models. With the help of symbolic computation package programms, we plot some grafical surfaces of them including high and lower points in a large range of independant variables. The results for the governing models are graphically introduced.

Keywords: (2+1)-dimensional asymmetrical Nizhnik-Novikov-Veselov equation, Sawada–Kotera (SK) equation, sine-Gordon expansion method, Hyperbolic, complex and mixed dark soliton solutions

1.INTRODUCTION

This survey investigates two different types models, firstly, the (2+1)-dimensional asymmetrical Nizhnik-Novikov-Veselov equation (ANNVE) defined as [1]

$$u_t + u_{xxx} + 3\left(u\int\left(u_x\right)dy\right)_x = 0, \tag{1}$$

which has been attracted attentions many experts from all over the world. J.G. Liu has presented several new Lump solutions to the ANNVE in [1]. Z.L. Zhao *et al* have introduced the various Lump type solutions of Eq.(1) [2]. M.S. Osman have studied on the fractional version for finding multiwave solutions of Eq.(1) [3]. Secondly, Sawada–Kotera equation (SKE) is given by [4]

$$u_t + 45u^2u_x - 15u_xu_{xx} - 15uu_{xxx} + u_{xxxxx} = 0, \tag{2}$$

where has been investigated extensively in the literature and called as fifth-order Sawada–Kotera equation along with its various forms. Eq.(2) has been investigated by many experts in terms of Lump solutions, exact soliton solutions, one-soliton solutions, periodic two-soliton solutions and singular periodic soliton solutions and many others by applying various methods such as inverse scattering, Hirota's bilinear method and so on. Furthermore, this model has been also investigated as [4]

$$5(\partial_x)^{-1}(u_{tt}) - 5u_{xxt} + 15uu_t - 15u_x(\partial_x)^{-1}(u_t) - 45u^2u_x - 15u_xu_{xx} - u_{xxxxx} = 0, \tag{3}$$

where $(\partial_x)^{-n} = \left(\frac{d}{d_x}\right)^{-n}$. Jianqing Lv and Sudao Bilige have investigated Eq.(3) known as the (2+1)-dimensional bidirectional Sawada–Kotera (bSK) equation [4]. SKE equation has received considerable attention over the last several decades. Several researches have been conducted in search of the exact solutions to the Sawada–Kotera equation. Y. Matsuno has successfully applied the bilinear transformation method in obtaining some periodic wave solutions in [5]. Zhang and Ma used the generalized bilinear forms in obtaining some Lump solutions to the (2+1)-dimensional Sawada–Kotera equation [6]. Sawada and Kotera introduced a method for Finding N-Soliton Solutions [7]. Liu and Dai presented exact soliton solutions for the fifth-order Sawada–Kotera equation to the literature and some important solitary wave solutions were reported [8]. Ramani observed the inverse scattering forms ordinary differential equations [9]. Therefor, many important nonlinear partial differential equations arising in real world problems have been investigated [16-33].

The rest of this paper is organized as follows: in Section 2, procedure of sine-Gordon expansion method (SGEM) for solving the ANNVE and SKE is given. in Section 3, many new complex hyperbolic and mixed dark soliton solutions are derived. Finally, some conclusions are given at the end of the paper.

2. THE FACTS OF THE SGEM

In this section, the brief description of the sine-Gordon expansion method is given. Considering the following equation of Sine Gordon [10-14]:

$$u_{xx} - u_{tt} = m^2 \sin(u) \tag{4}$$

where $u=u(x,t)$ and m is a real constant. Using of the wave transformation $u=u(x,t)=U(\xi),\quad \xi=\mu(x-ct)$ to Eq. (4) produces the following ordinary non-linear differential equations (NODE) as

$$U^{"}=\frac{m^2}{\mu^2\left(1-c^2\right)}\sin(U) \tag{5}$$

where $U=U(\xi),\xi$ is the amplitude of the wave and c is the location of the wave. We can write its complete simplification Eq. (5) as:

$$\left[\frac{U^{'}}{2}\right]^2=\frac{m^2}{\mu^2\left(1-c^2\right)}\sin^2\left(\frac{U}{2}\right)+K \tag{6}$$

Substituting $K=0$, $w(\xi)=\frac{U^{'}}{2}$ and $a^2=\frac{m^2}{\mu^2\left(1-c^2\right)}$ into Eq. (6), we get

$$w'(\xi)=a\sin(w) \tag{7}$$

laying $a=1$, Eq. (7) becomes;

$$w'(\xi)=\sin(w) \tag{8}$$

the following two important equations are acquired from Eq. (8);

$$\sin(w)=\sin(w(\xi))=\left.\frac{2pe^{\xi}}{p^2e^{2\xi}+1}\right|_{p=1}=\operatorname{sech}(\xi) \tag{9}$$

$$\cos(w)=\sin(w(\xi))=\left.\frac{p^2e^{2\xi}-1}{p^2e^{2\xi}+1}\right|_{p=1}=\tanh(\xi) \tag{10}$$

where p is the integral constant. Consider the general form of nonlinear partial differential equations in two independent variables x and t

$$P(u,u_x,u_t,\ldots)=0, \tag{11}$$

where $u_x=\frac{\partial u(x,t)}{\partial x}$ is the derivative of $u(x,t)$ with respect to x; $u=u(x,t)$ is an unknown function, P is a polynomial in $u=u(x,t)$ and its partial derivatives, in which the nonlinear terms and highest order derivatives are involved. Substituting the following traveling wave transformation into Eq. (8):

$$u=u(x,t)=U(\xi),\quad \xi=\mu(x-ct), \tag{11a}$$

where μ and c are arbitrary constants, we get the following nonlinear ordinary differential equation (NODE):

$$N\left(U,U',U'',U''',\cdots\right)=0,$$

where N is a polynomial of $U=U\left(\xi\right)$ and its derivatives and the superscripts indicate the ordinary derivatives with respect to ξ. The following steps are further followed.
Assume the solution of $U=U\left(\xi\right)$ to be as

$$U\left(\xi\right)=\sum_{i=1}^{n}\tanh^{-1}\left(\xi\right)\left[B_i\,\mathrm{sech}\left(\xi\right)+A_i\tanh\left(\xi\right)\right]+A_0 \tag{12}$$

Eq. (12) can be rewritten by Eq. (9) and (10) as follows:

$$U\left(w\right)=\sum_{i=1}^{n}\cos^{-1}\left(\xi\right)\left[B_i\sin\left(w\right)+A_i\cos\left(w\right)\right]+A_0. \tag{13}$$

where the unknown parameters $A_0, A_n, B_n;\ \left(i=1,2,3,\cdots,n\right)$ are identified later. Using the balance principle, identifying the value of n by taking into account the highest power of nonlinear term and the highest derivative in NODE is obtained. Letting the coefficients of $\sin^i\left(w\right),\cos^i\left(w\right)$ to be all zero, obtain a system of equations, solving these equations by using one ofnumerical computation programme, gives the values of A_i, B_i, μ and c. Finally, substituting the values of A_i, B_i, μ and c into Eq. (13), we obtain the new traveling wave solutions to Eq. (11).

3. Applications of SGEM to the models

In this section of paper, we use SGEM to find new complex singular soliton solutions to the (2+1)-dimensional asymmetrical Nizhnik-Novikov-Veselov and Sawada–Kotera equations.

3.1 Applications of SGEM to the ANNVE

Firstly, in this subsection, we apply the sine-Gordon equation method to acquire some new complex solitary wave solutions for the (2+1)-dimensional asymmetrical Nizhnik-Novikov-Veselov equation. Substituting $u\left(x,y,t\right)=U\left(\xi\right), \xi=kx+ry-ct,$ into Eq.(1), we have the following NODE:

$$rk^3U''-rcU+3k^2U^2=0, \tag{14}$$

where r,c and k are non-zero real numbers. Balancing the terms U'' and U^2, we get $n=2$. Considering $n=2$ in Eq.(13), we find the solution form as

$$U\left(w\right)=B_1\sin\left(w\right)+A_1\cos\left(w\right)+B_2\cos\left(w\right)\sin\left(w\right)+A_2 cos\left(w\right)^2+A_0, \tag{15}$$

where A_0, A_1, B_1, A_2 and B_2 are constants. Putting Eq.(15) and its second derivation into Eq.(14), getting a system of trigonometric function, solving this system by some computational programs such as Matlab and Mathematica, gives new hyperbolic solutions and complex solutions, as follows:

Case1: With $A_0 = kr, A_1 = 0, A_2 = -kr, B_1 = 0, B_2 = ikr, c = k^3$, we get

$$u_1(x,t) = kr\operatorname{sech}\left(k^3t - kx - ry\right)\left(\operatorname{sech}\left(k^3t - kx - ry\right) - i\tanh\left(k^3t - kx - ry\right)\right), \tag{16}$$

which k and r real constants with non-zero.

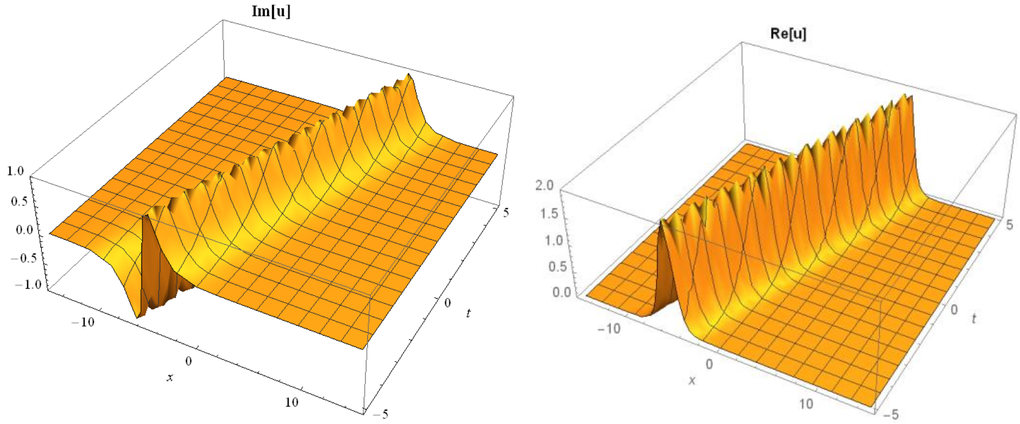

Fig.1. Three-dimensional graph of solution of $u_1(x,t)$ by considering the values $k = 1, r = -2, y = 0.2.$

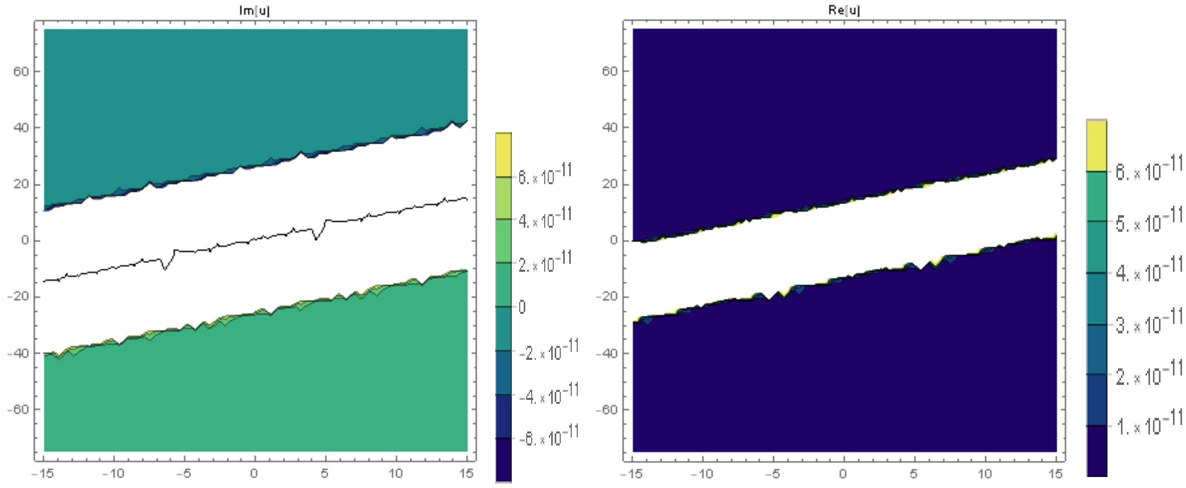

Fig.2. Contour graph of solution of $u_1(x,t)$ by considering the values $k = 1, r = -2, y = 0.2.$

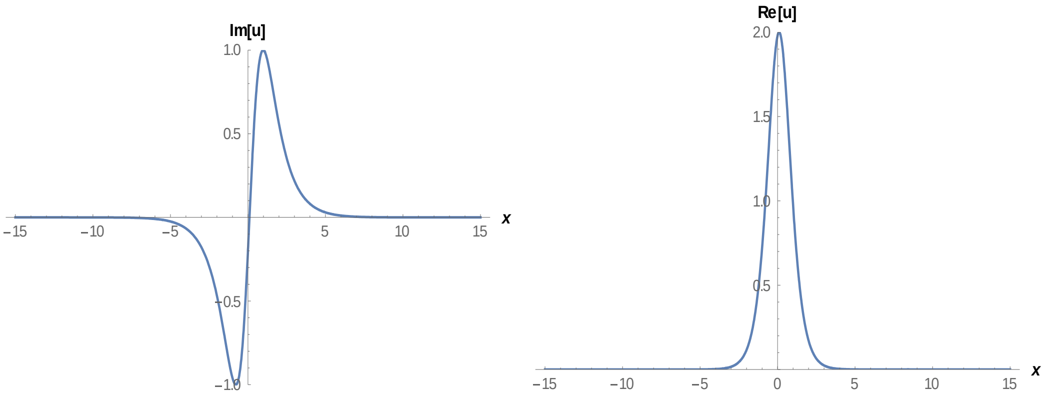

Fig.3. Two-dimensional graph of solution of $u_1(x,t)$ by considering the values $k = 1, r = -2, y = 0.2, t = 0.5.$

Case 2: When $A_0 = kr, A_1 = 0, A_2 = -kr, B_1 = 0, B_2 = -ikr, c = k^3$, we find

$$u_2(x,t) = kr\operatorname{sech}\left(k^3t - kx - ry\right)\left(\operatorname{sech}\left(k^3t - kx - ry\right) + i\tanh\left(k^3t - kx - ry\right)\right), \tag{16}$$

which k and r real constants with non-zero.

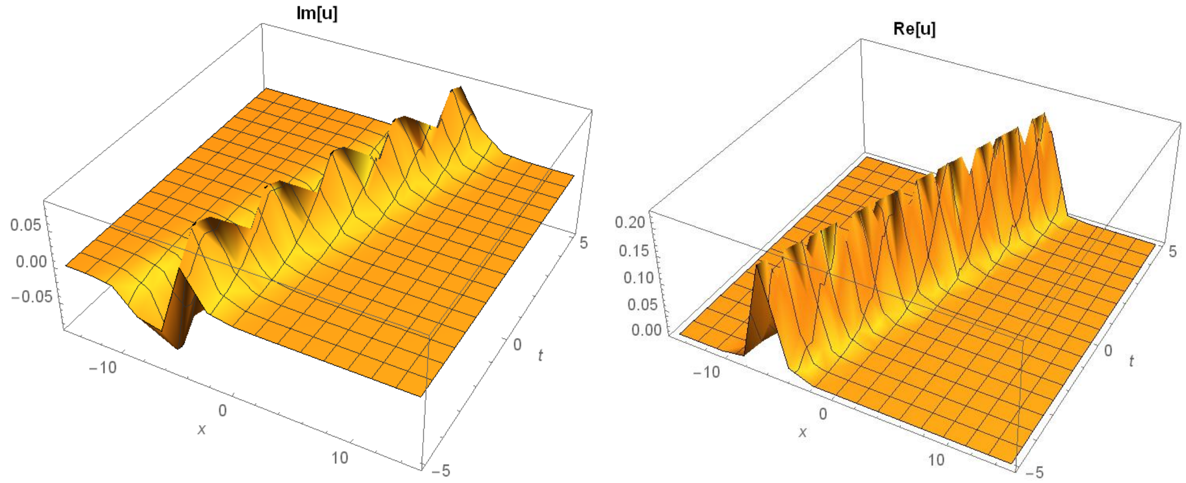

Fig.4. Three-dimensional graph of solution of $u_2(x,t)$ by considering the values $k = -1, r = -0.2, y = 6.$

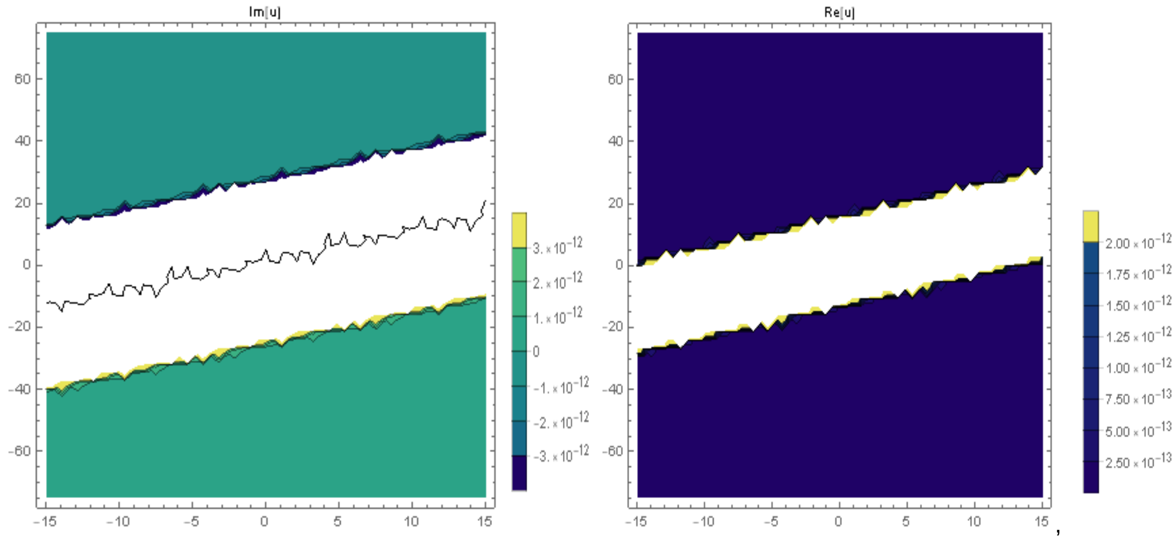

Fig.5. Contour graph of solution of $u_2(x,t)$ by considering the values $k = -1, r = -0.2, y = 6.$

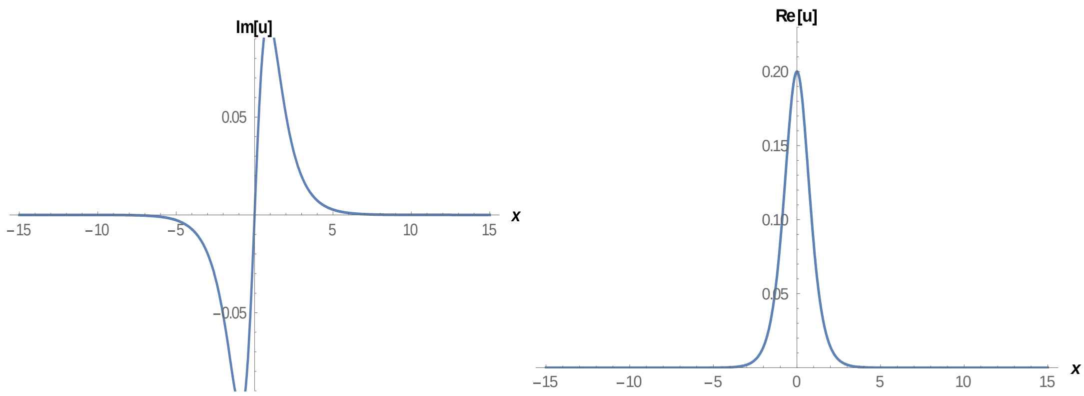

Fig.6. Two-dimensional graph of solution of $u_2(x,t)$ by considering the values

$k = -1, r = -0.2, y = 6, t = 0.1.$

3.2 Applications of SGEM to the SKE

Secondly, we apply SGEM to find some new complex singular soliton solutions to the Sawada–Kotera equation. Putting $u=u(x,t)=U(\xi),\xi=kx-ct$ into Eq.(2), we have the following NODE:

$$k^5U^{(4)}-15k^3U''U-cU+15kU^3=0 \tag{17}$$

where k,c are non-zero real constants. Balancing the terms $U^{(4)}$ and $U''U$, we get $n=2$. producing

$$U(w)=B_1\sin(w)+A_1\cos(w)+B_2\cos(w)\sin(w)+A_2cos(w)^2+A_0, \tag{18}$$

where A_0,A_1,B_1,A_2 and B_2 are constants. Putting Eq.(18) and its various derivations into Eq.(17), getting a system of trigonometric function, solving this system by some computational programs such as Matlab and Mathematica, gives new complex singular mixed dark soliton solutions as follows:

Case 3.2.1: If $A_0=-k^2,A_1=0,A_2=k^2,B_1=0,B_2=ik^2,c=k^5$, produces

$$u_3(x,t)=-k^2+ik^2\sec h(k^5t-kx)\tan h(k^5t-kx)+k^2\tanh(k^5t-kx), \tag{19}$$

which k is real constant with non-zero.

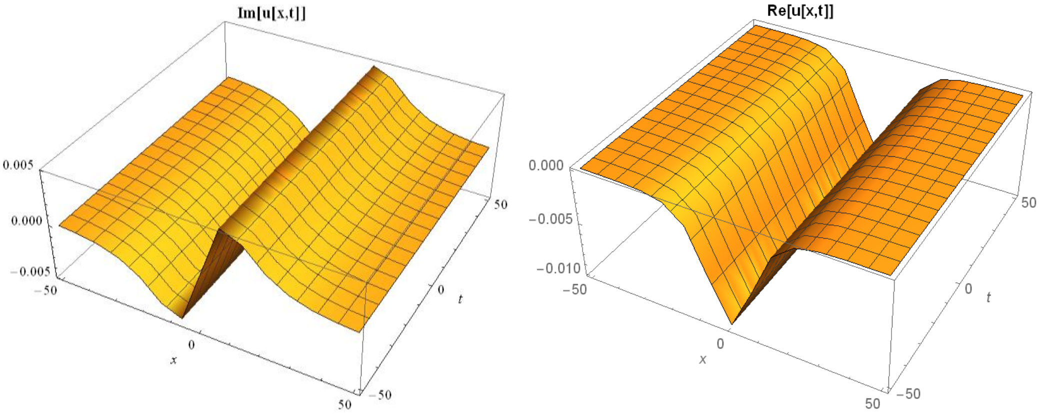

Fig.7. Three-dimensional graph of solution of $u_3(x,t)$ by considering the value of $k=-0.1$.

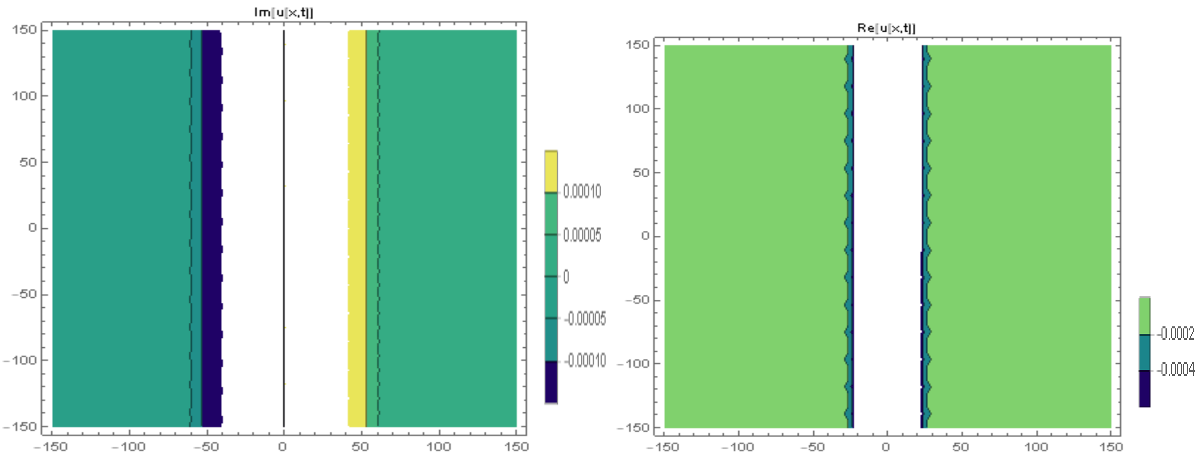

Fig.8. Contour graph of solution of $u_3(x,t)$ by considering the value of $k=-0.1$.

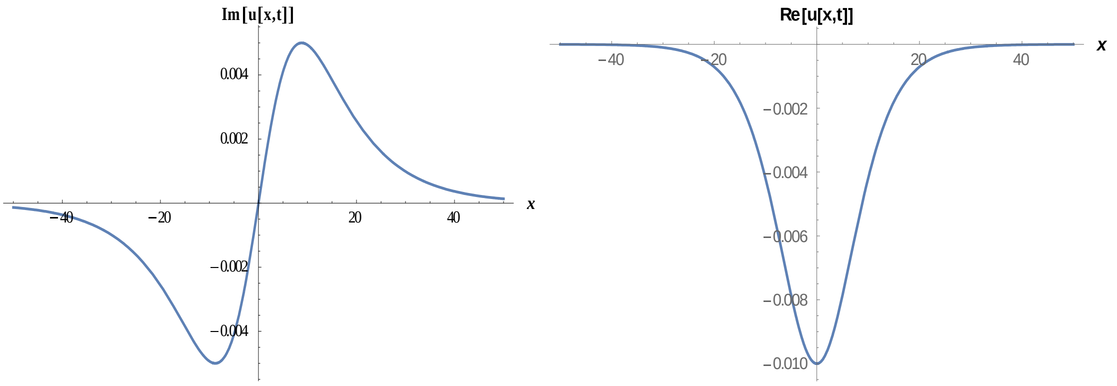

Fig.9. Two-dimensional graph of solution of $u_3(x,t)$ by considering the values $k=-0.1, t=0.1.$

Case 3.2.2: If we consider the following coefficients,

$$A_0=-\frac{1}{60}\left(\frac{1}{2}\left(11+\sqrt{105}\right)\right)^{2/5}\left(45+\sqrt{105}\right)c^{2/5}, A_1=0, B_1=0,$$

$$A_2=\left(\frac{1}{2}\left(11+\sqrt{105}\right)\right)^{2/5}c^{2/5}, B_2=i\left(\frac{1}{2}\left(11+\sqrt{105}\right)\right)^{2/5}c^{2/5}, k=\left(\frac{1}{2}\left(11+\sqrt{105}\right)\right)^{1/5}c^{1/5},$$ we can

find new complex mixed dark singular soliton solution as

$$\begin{aligned} u_4(x,t)=&-\frac{1}{60}\left(\frac{1}{2}\left(11+\sqrt{105}\right)\right)^{2/5}\left(45+\sqrt{105}\right)c^{2/5}-i\left(\frac{1}{2}\left(11+\sqrt{105}\right)\right)^{2/5}c^{2/5}\\ &\times\operatorname{sec}h\left(ct-\left(\frac{1}{2}\left(11+\sqrt{105}\right)\right)^{1/5}c^{1/5}x\right)\tan h\left(ct-\left(\frac{1}{2}\left(11+\sqrt{105}\right)\right)^{1/5}c^{1/5}x\right)\\ &+\left(\frac{1}{2}\left(11+\sqrt{105}\right)\right)^{2/5}c^{2/5}\tanh^2\left(ct-\left(\frac{1}{2}\left(11+\sqrt{105}\right)\right)^{1/5}c^{1/5}x\right),\end{aligned} \tag{20}$$

which c is real constant with non-zero. Under the suitable values of parameters, some simulations of this result can be observed as;

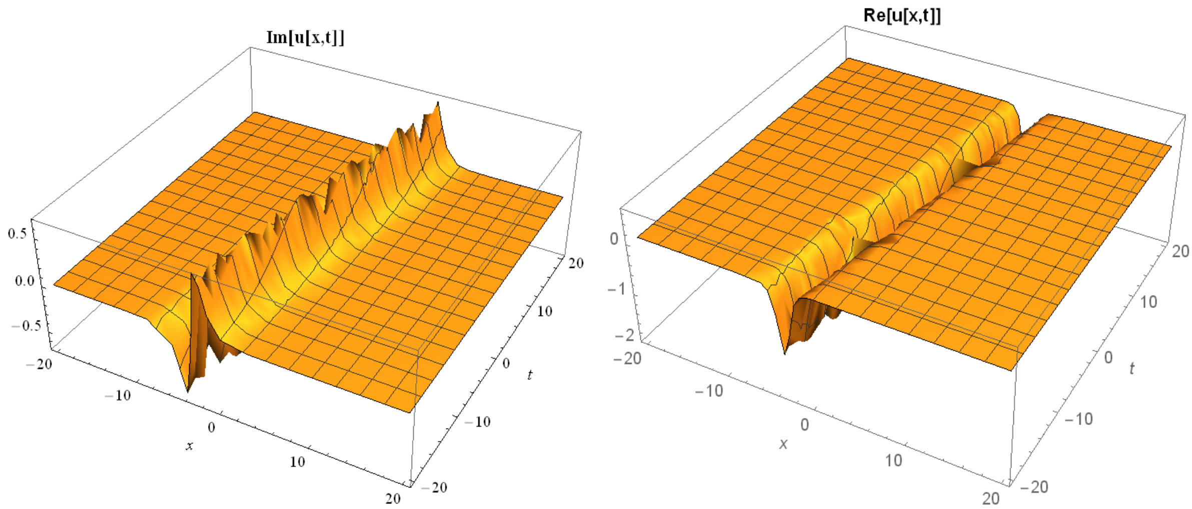

Fig.10. Three-dimensional graph of solution of $u_4(x,t)$ by considering the value of $c=0.2.$

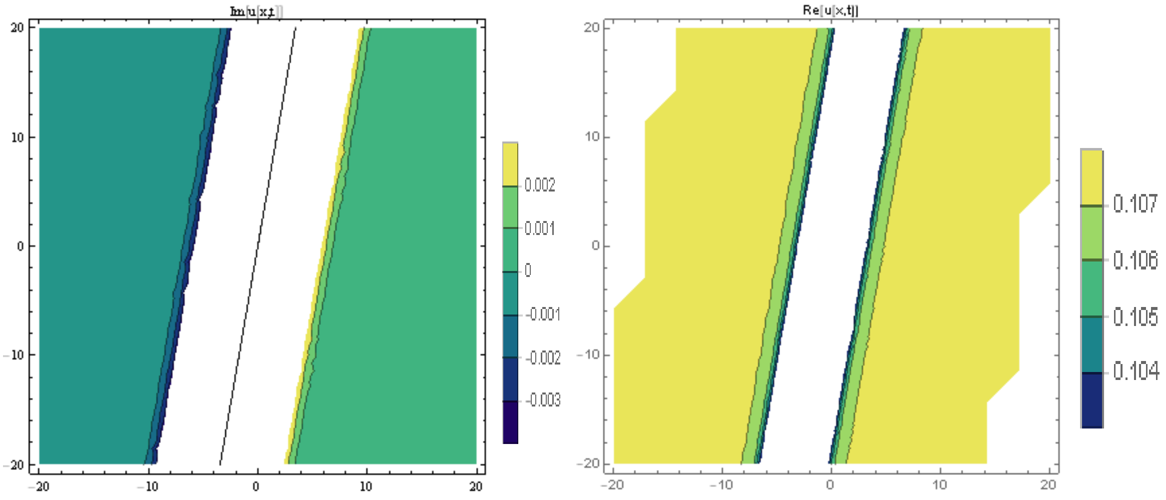

Fig.11. Contour graph of solution of $u_4(x,t)$ by considering the value of $c = 0.2$.

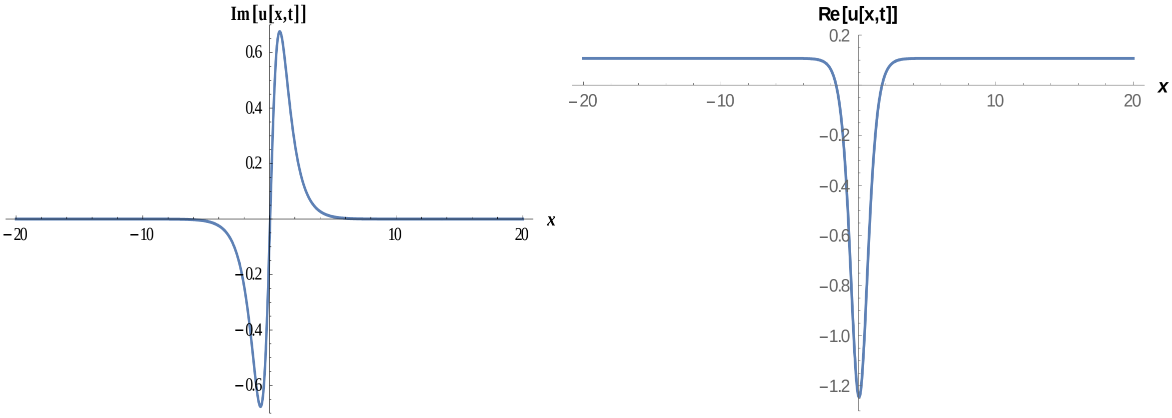

Fig.12. Two-dimensional graph of solution of $u_3(x,t)$ by considering the values $k = -0.1, t = 0.1$.

4. Conclusion

In this paper, we have investigated two nonlinear partial differential equations, namely, (2+1)-dimensional asymmetrical Nizhnik-Novikov-Veselov and Sawada–Kotera (SK) equations. Via SGEM, variety of new solitary wave solutions with the complex hyperbolic and mixed dark singular soliton structure have been successfully derived. For better understanding the physical importance of the founding solutions in this study, we have presented various graphiycal simulations in two- and three-dimensional graphs with suitable values of the arbitrary constants. The results for the governing models are graphically introduced. When we compare these results with the existing solutions, it can be observed that these are entirely new complex hyperbolic and mixed dark singular soliton solutions.

Such reported solutions have some important physical meaning, for instance; the hyperbolic tangent arises in the calculation of magnetic moment and rapidity of special relativity, the hyperbolic cotangent arises in the Langevin function for magnetic polarization and the hyperbolic secant arises in the profile of a laminar jet [15] . Therefore, it is estimated that these results are of similar important physical meanings. The SGEM is a powerful anaytical schemes that gives good results when applied to various nonlinear evolution equations. To the best of our knowledge, applications of SGEM to the ANNVE and SKE equations have not been submitted to the literature beforehand.

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