Investigation of Two-Dimensional Viscoelastic Fluid with Non-Uniform Heat Generation Over Permeable Stretching Sheet with Slip Condition

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Abstract

Here in this research article we have investigated incompressible viscoelastic fluid flow over a uniform stretching surface sheet along with slip boundary conditions in the presence of porous media. The partial differential which govern the fluid flow are changed into ordinary differential equations through suitable similarity transformation variables Finally the transformed ordinary differential equations are solved with the help of semi numerical technique known as homotopy analysis method (HAM). The uniqueness of our study is not only to analyzed and carried out the effect of elastic parameter, but also to account for various dissipation which is important in the case of optically transparent flow. The novel effects for the parameters which affect the flow and heat-transfer, such as the Eckert number, porous medium parameter and the velocity slip parameter are studied through graphs. Also, the convergence analysis for the proposed method is addressed. Additionally, for the sake of validation present work is also compared with already published work and outstanding agreement is found.

Keywords: viscoelastic fluid, porous medium, slip velocity, analytical approach.

1. Introduction

In recent years, gradual development in fluid dynamics, the flow over a stretching surface has resulted in active studies, due to its practical applications such as hot rolling, fiber plating and lubrication porous. Crane [1], first introduced an analytical solution of Newtonian boundary-layer flow due to a stretching surface. Vleggar [2] studied the laminar flow of Newtonian fluid on continuous accelerating stretching surface. Dutta et al. [3] investigated the temperature field flow due to a stretching sheet with uniform heat flux. In the content, a similar problem of Newtonian fluid flow due to the stretching surface have been investigated by many researchers [4, 5].

Investigation of viscoelastic fluid over a continuous stretching surface finds many important applications in the fields of engineering fluid mechanics such as inks, paints, jet fuels, polymer extrusion, drawing of plastic fiber and wire. The over increasing applications to this type of fluid, many researchers turned to the study of this type under different situations. Vajravelu and Rollins [6] studied viscoelastic fluid over a stretching surface with effect of heat transfer. Andersson [7] analyzed the effect of MHD on viscoelastic fluid flow due to stretching surface. Incompressible flow of viscoelastic fluid and heat transfer over a stretching sheet embedded in a porous medium has been investigated by Suhas and Veena [8]. Viscoelastic boundary layer fluid flow and hear transfer over an exponential stretching sheet has been discussed by Sanjaya and Khan [9]. Naneppanavar [10] studied the flow and heat transfer characteristic of a viscoelastic fluid over an impermeable stretching sheet embedded in a porous medium with viscous dissipation and heat transfer.

In the above studies the effect of velocity slip is absent. This phenomenon is very important in fluid mechanics. It was first introduced by Navier [11]. Thompson and Troian studied [12] studied the incompressible flow at solid surface with general boundary condition. Slip effects and heat transfer analysis in a viscos fluid over an oscillatory stretching surface has been studied by Abba et al. [13]. MHD slip flow of viscoelastic fluid over stretching surface has been investigated by Turkyilmazoglu [14]. Ferras et al. [15] analyzed slip flow of Newtonian and viscoelastic fluids. The effect of slip and MHD on viscoelastic convection flow in a vertical channel has been discussed by Singh [16]. Krishan [17] analyzed magnetohydrodynamics mixed convection viscoelastic slip flow through a porous medium in a vertical porous channel with thermal radiation. The effect of slip conditions on the peristaltic flow of a Jeffrey fluid with a Newtonian fluid is studied by Vajravelu et al. [18].

For the non-Newtonian fluids, the perdition of heat transfer analysis is very important due to its practical engineering uses, such as food-processing, flow through filtering media and oil recovery. Because of the above motivation, in the present work, a new visualization for the effects of the non-uniform heat generation/ absorption, velocity slip and viscous dissipation with heat transfer flow of viscoelastic fluid due to stretching surface embedded in a porous medium is analyzed. Recently, viscoelastic Oldroyd 8-constant fluid has been analyzed for wire coating by Zeeshan et.al [19] using the Runge-Kutta method with heat transfer effect. Prasad et al. [20] investigated magnetohydrodynamic mixed convicted heat flow over a nonlinear sheet with temperature-dependent viscosity. Similarly, Awati [21] carried out an analysis of MHD viscous flow with a heat source. Series and analytical solution have been obtained and the effects of emerging parameters were discussed through graphs. Ahmad et al. [22] investigated a steady flow of a power law fluid through an artery with a stenosis has been studied and the effects of various parameters of interest discussed through graphs. A detail analysis of MHD flow and heat transfer through viscoelastic fluid in presence of porous medium in wire coating analysis has been carried out by Zeeshan et al. [23]

In the present study, two-dimensional flow of viscoelastic fluid with non-uniform heat source generation along a permeable stretching sheet is investigated analytically by semi-analytical method HAM with slip conditions. The modeled partial differential equations are converted to ordinary differential equations by using similarity variables. The series solutions have been obtained by HAM. The effect of emerging parameters involved in the solution has been discussed through graphs in detail. Additionally, for the accuracy of the results the present work is also compared with the published work of Rajagopal et al. [24].

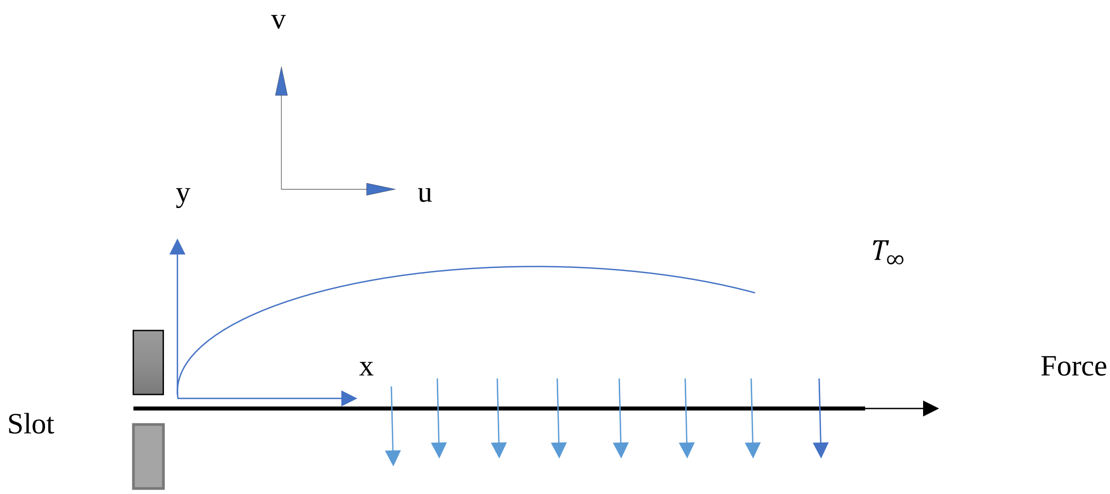

Fig.1. Physical Configurations Diagram

2. Formulation of the problem

In this section, we will consider a two-dimensional boundary layer flow of an incompressible viscoelastic fluid over a stretching sheet embedded in a porous medium. The origin is located at a slit, through which the sheet (see Figure 1) is drawn through the fluid medium. The x-axis is chosen along with the sheet and the y-axis is taken normal to it. The sheet is assumed to have the velocity $u = cx$ where x is the coordinate measured along the stretching surface and $C > 0$ is a constant for a stretching sheet. Likewise, the temperature distribution for the sheet is assumed to be in the form $T_w = T_\infty + Ax^r$ where T_w is the temperature of the sheet, T_∞ is the temperature of the ambient, A and r are constants. Also, the sheet is assumed to be porous with the suction velocity v_w. Making the usual boundary layer approximations the boundary layer equations read

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial x^2} - \frac{\mu_e}{\rho k}u - \frac{k_0}{\rho}\left(u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{k}{\rho c_p}\left(\frac{\mu_e}{k}u^2+u\left(\frac{\partial u}{\partial x}\right)^2\right)+\frac{q'''}{\rho c_p}, \quad (3)$$

where u and v are the velocity components in the x and y directions, respectively. ρ is the density of the fluid, κ is the fluid thermal conductivity and K_0 is a positive parameter associated with the viscoelastic fluid. T is the temperature of the fluid, μ is the fluid viscosity, μ_e is the dynamic viscosity of the fluid due to the flow in the porous medium, k is the permeability of the porous medium, q''' is the rate of internal heat generation, and c_p is the specific heat at constant pressure. We must observe that in the second term of the right-hand side of equation (3), we follow [25–28].

The boundary conditions with the slip condition [18–20] can be written as

$$u=U+a\left(\frac{\partial u}{\partial y}-\frac{k_0}{\mu}\left(u\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^2 u}{\partial y^2}+2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\right)\right), \quad (4)$$

$$\begin{aligned} &v=-v_w,\ T_w=T_\infty+Ax^r\ \ at\ y=0,\\ &u\to 0,\ T_w\to T_\infty,\ as\ y\to\infty, \end{aligned} \quad (5)$$

where a is the velocity slip factor, the mathematical analysis of the problem is simplified by introducing the following dimensionless coordinates.

$$\eta=y\sqrt{\frac{c}{\upsilon}},\ u=cxf'(\eta),\ v=-\sqrt{c\upsilon}\,f(\eta), \quad (6)$$

$$\theta(\eta)=\left(\frac{T-T_\infty}{T_w-T_\infty}\right), \quad (7)$$

where η is the similarity variable, $f(\eta)$ is the dimensionless stream function, $v=\frac{\mu}{\rho}$ is the kinematic viscosity, and $\theta(\eta)$ is the dimensionless temperature. It can be seen that a similarity solution exists only when we take $r=2$. Likewise, the internal heat generation or absorption q''' is modeled according to the following formula [29].

$$q'''=\left(\frac{kU}{vx}\right)\left[a^*\left(T-T_\infty\right)e^{-\eta}+b^*\left(T_w-T_\infty\right)\right]. \quad (8)$$

Therefore, upon using these variables, the boundary layer governing equations (1)-(3) can be written in the following non-dimensional form:

$$f'''-(f')^2+ff''-\beta f'+k\left((f'')^2-2f'f'''+ff''''\right)=0, \tag{9}$$

$$\frac{1}{\Pr}\theta''+f\theta'-2f'\theta+Ec\left(\beta(f')^2+(f'')^2\right)+\frac{1}{\Pr}\left(a^*e^{-\eta}+b^*\theta\right)=0, \tag{10}$$

the boundary conditions are

$$f=f_w,\ f=1+\lambda\left[(1+3kf')f''+kf_wf'''\right],\ \theta=1,\ at\ \eta=\infty, \tag{11}$$

$$f'\to 0,\ f''\to 0,\ \theta\to 0,\ at\ \eta\to\infty, \tag{12}$$

where $\beta=\frac{\mu_e}{\rho ck}$ is the porous parameter, $K=\frac{ck_0}{\mu}$ is the viscoelastic parameter, $\Pr=\frac{\mu c_p}{\kappa}$ κ is the Prandtl number, $Ec=\frac{c^2}{Ac_p}$ is the Eckert number, $f_w=\frac{v_w}{\sqrt{cv}}>0$ is the suction velocity parameter, and $\lambda=a\sqrt{\frac{c}{v}}$ is the velocity slip parameter.

3. HAM Solution

In order to solve equations (9) and (10) under the boundary conditions (11) and (12), we utilize the homotopy analysis method with the following procedure. The solutions having the auxiliary parameters $\hbar$ regulate and control the convergence of the solutions. The initial guesses are selected as follows:

We select the initial approximations such that the boundary conditions are satisfied as follows:

$$f_0(\eta)=s-1+e^{-\eta} \text{ and } \theta_0(\eta)=e^{-\eta}. \tag{13}$$

The linear operators are introduced as $\Im_f$ and $\Im_\theta$:

$$\Im_f(f)=f''' \text{ and } \Im_\theta(\theta)=\theta''. \tag{14}$$

With the following properties:

$$\Im_f(c_1+c_2\eta+c_3\eta^2+c_4e^{-\eta})=0 \text{ and } \Im_\theta(c_5+c_6e^{-\eta})=0, \tag{15}$$

where $c_i(i=1-6)$ are arbitrary constants in general solution.
The nonlinear operators, according to (9) and (10), are defined as:

$$\aleph_f\left[f(\eta;p)\right]=\frac{\partial^3 f(\eta;p)}{\partial\eta^3}-\left(\frac{\partial f(\eta;p)}{\partial\eta}\right)^2+f(\eta;p)\frac{\partial^2 f(\eta;p)}{\partial^2\eta}-\beta\frac{\partial f(\eta;p)}{\partial\eta}+$$

$$\kappa\left(\left(\frac{\partial^2 f(\eta;p)}{\partial^2\eta}\right)^2-2\frac{\partial f(\eta;p)}{\partial\eta}\frac{\partial^3 f(\eta;p)}{\partial\eta^3}+f(\eta;p)\frac{\partial^3 f(\eta;p)}{\partial\eta^3}\right)=0, \quad (16)$$

$$\aleph_\theta\left[f(\eta;p),\theta(\eta;p)\right]=\frac{1}{\Pr}\frac{\partial^2\Theta(\eta;p)}{\partial\eta^2}+\Pr f(\eta;p)\frac{\partial\Theta(\eta;p)}{\partial\eta}-2f(\eta;p)\frac{\partial\Theta(\eta;p)}{\partial\eta}+$$

$$Ec\left(\beta\left(\frac{\partial\Theta(\eta;p)}{\partial\eta}\right)^2+\left(\frac{\partial^2\Theta(\eta;p)}{\partial\eta^2}\right)^2\right)+\frac{1}{\Pr}\left(a^*e^{-\eta}+b^*\Theta\right)=0$$

The auxiliary function become,

$$H_f(\eta)=H_\theta(\eta)=e^{-\eta}. \quad (17)$$

The symbolic software Mathematica is employed to solve *i*th order deformation equations:

$$\Im_f\left[f_i(\eta)-\chi_i f_{i-1}(\eta)\right]=\hbar_f\mathcal{H}_f\left[f(\eta)\right]R_{f,i}(\eta),$$

$$\Im_\theta\left[\theta_i(\eta)-\chi_i\theta_{i-1}(\eta)\right]=\hbar_\theta\mathcal{H}_\theta(\eta)R_{\theta,i}, \quad (18)$$

where $\hbar$ is auxiliary non-zero parameter and

$$R_{f,i}(\eta)=f'''_{m-1}-\sum_{k=0}^{m-1}f'_{m-1-k}f'_k+\sum_{k=0}^{m-1}f_{m-1-k}f'_k-\beta f'_{m-1}+k\left(\sum_{k=0}^{m-1}f'_{m-1-k}f'_k-2f'_{m-1}\sum_{k=0}^{m-1}f'_{m-1-k}f'_k+\sum_{k=0}^{m-1}f'_{m-1-k}\sum_{l=0}^{k}f'_{k-1}f'''_l\right) \quad (19)$$

$$R_{\theta,i}(\eta)=\frac{1}{\Pr}\theta''_{m-1}+\sum_{k=0}^{m-1}f_{m-1-k}\theta'_k-2\sum_{k=0}^{m-1}\theta_{m-1-k}f'_k+Ec\left(\beta\sum_{k=0}^{m-1}f_{m-1-k}f'_k+\sum_{k=0}^{m-1}f_{m-1-k}f''_k\right)+\frac{1}{\Pr}\left(a^*e^{-\eta}+b^*\theta\right)$$

$$\chi_i=\begin{cases}0, & \text{if } i\le 1\\ 1, & \text{if } i>1\end{cases},$$

are the involved parameters in HAM theory. For more details about the theory of Homotopy Analysis Method see [30-40].

3.1. Convergence of the method

To validate the method, the convergence of the method is also, necessary. For this purpose, h-curve has been drawn which ensure the convergence of the series solution. The calculations are carried out on a personal computer with 4GB RAM and 2.70 GHz CPU. The code is developed using computer software Mathematica Zeeshan et at. [41]. To see the range of admissible values of these parameters of the and are plotted in Fig.2 and Fig.3 given by 20th order approximation which takes approximately less than a minute in execution. The suitable range for h_f and h_θ are $-1.5\le h_f\le -0.3$ and $-1.7\le h_\theta\le -0.3$, respectively.

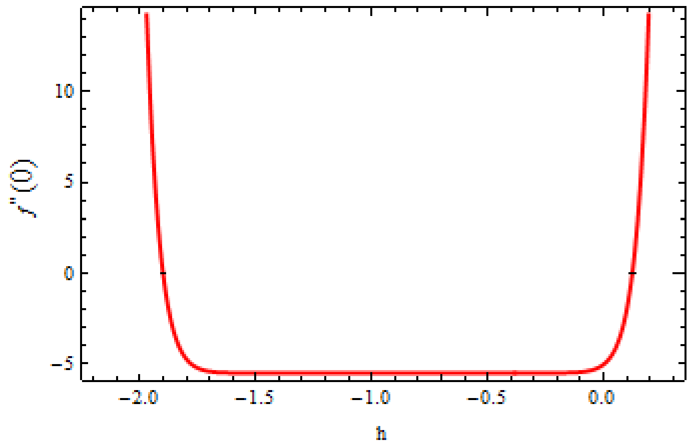

Figure 2. h-curve for velocity field.

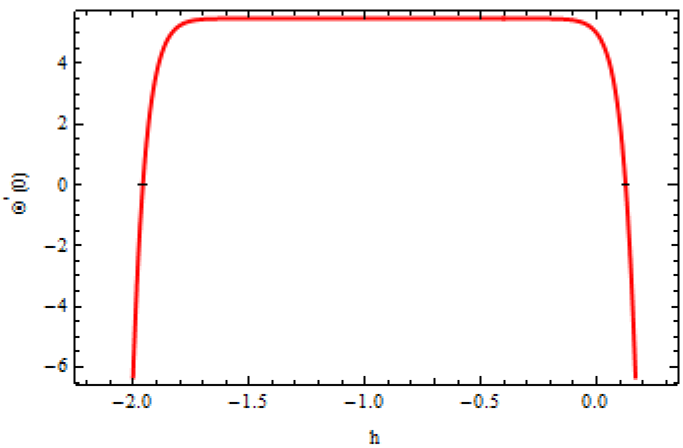

Fig.3. h-curve for temperature field.

4. Results and discussion

Two-dimensional non-Newtonian viscoelastic fluid with non-uniform heat generation over a permeable stretching sheet embedded in a porous medium has been investigated. The similarity transformation has been applied to transform the PDEs to ODEs. The analytical solution has been obtained by using HAM. For the validation of our analytical solution, a comparison has been done with the published work of Rajagopal at el. [24]. This comparison is given in table-I. This ensures that our results are excellent and agreement. The computation results are displayed in figures 4-13.

From figure 4, it is observed that the velocity of the fluid decreases with the increasing values of porous parameter β. Physically increases in β mean a high dynamic viscosity μ_e, which corresponds to porous medium and a small permeability for the porous medium, which causes the production of the resistance force to the fluid flow which causes a decrease for the velocity distribution enhances along the boundary layer as depicted in figure 5. Also, from this figure it is clear that with increasing of porous parameter, the thermal boundary layer becomes thicker but the momentum boundary layer becomes thinner.

The effect of Eckert number E_c on velocity and temperature profiles is shown in figure 6 and 7 respectively. From figure 6, we see that the velocity curve lower when the Eckert number is larger and so, the momentum, effect is lower. Also, from figure 7 we notice that the thermal boundary layer becomes thicker when the Eckert number increases but the temperature distribution enhances.

Figure 8 and 9 are plotted to see the effect of slip parameter versus similarity variable η on velocity and temperature profiles. It is investigated that the velocity of the fluid decreases with the increasing values of the slip velocity parameter while with the increases of the same parameter the temperature is increased.

The effect of suction parameter f_w on the fluid flow and temperature profile has been analyzed and results are given in figure 10 and 11 respectively. These figures show that the suction parameter has significant effect on the boundary layer thickness. The suction parameter reduces the boundary layer thickness as a result the fluid flow and the temperature distribution reduce.

The effect of internal heat generation parameters on the thermal boundary layer thickness are presented in figures 12 and 13. It is observed that as the values of the internal heat generation parameters $a > 0 \text{ and } b > 0$ become stronger, the thermal boundary layer thickness increases, whereas the internal heat generation parameters $a < 0 \text{ and } b < 0$ have the opposite effect. Also, it's noticed that the highest temperature distribution for the fluid in the boundary layer was obtained with the greatest heat generation parameters $a^* > 0 \text{ and } b^* > 0$. Likewise, it is shown that the effect of the heat absorption parameters $a^* < 0 \text{ and } b^* < 0$ causes a drop in the temperature distribution as the heat following from the sheet is absorbed.

At last for accuracy of the problem, the present work is also compared with the published work reported by Rajagopal et al. [24] and outstanding agreements founded and also, clarified from table 1.

Table-1. Comparison of the present work with published work

	Rajagopal [24]	Present work
0.0	0.98561340	0.98561423
1.0	0.27908819	0.27908734
2.0	0.09291179	0.09291328
3.0	0.03295374	0.03295452
4.0	0.01196183	0.01196265

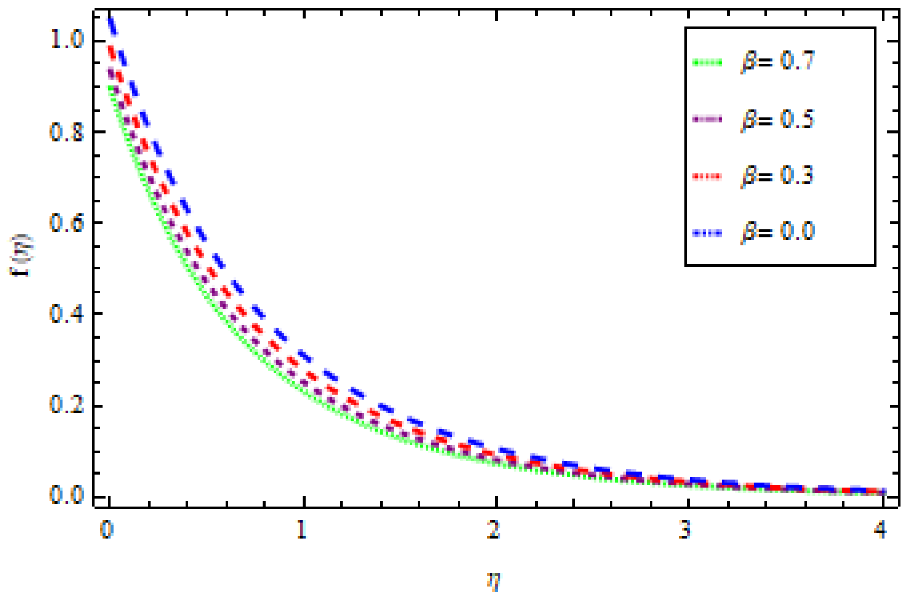

Figure 4. The influence of the velocity profile for different values of β, when $K = 0.1,\ fw = 0.3,\ \lambda = 0.2,\ \Pr = 5.0,\ Ec = 0.4,\ a^* = b^* = 0.2.$

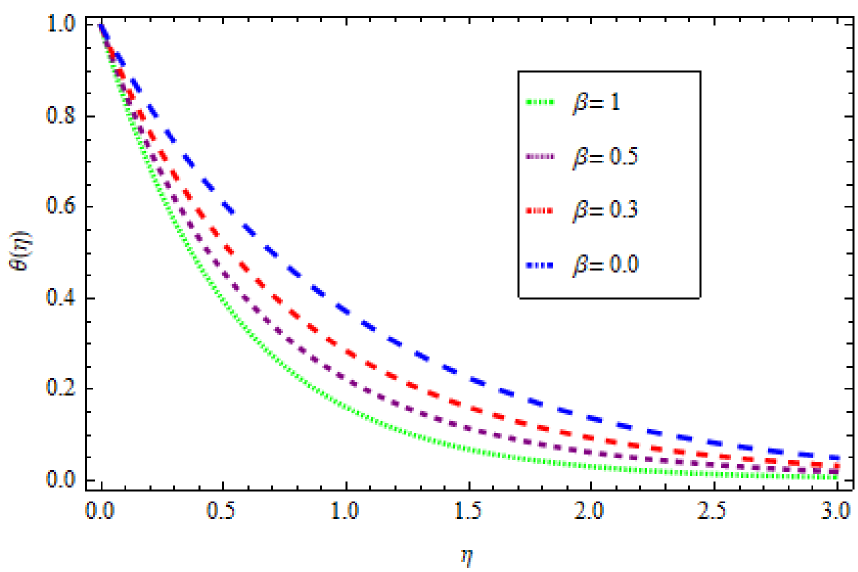

Figure 5. The influence of the temperature profile for different values of β, when $K = 0.1,\ fw = 0.3,\ \lambda = 0.2,\ \Pr = 5.0,\ Ec = 0.4,\ a^* = b^* = 0.2.$

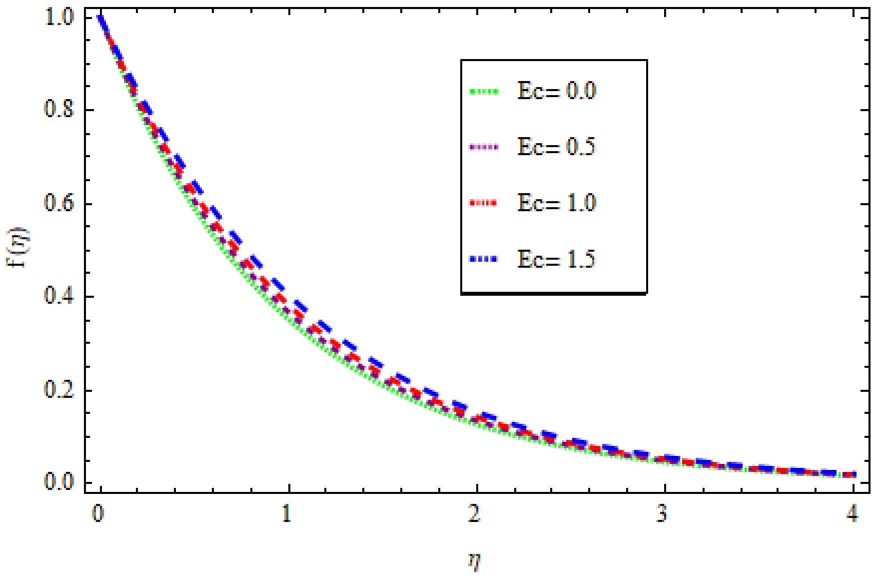

Figure 6. The influence of the velocity profile for different values of E_c, when $K = 0.1,\ fw = 0.3,\ \lambda = 0.2,\ \Pr = 5.0,\ \beta = 0.4,\ a^* = b^* = 0.2.$

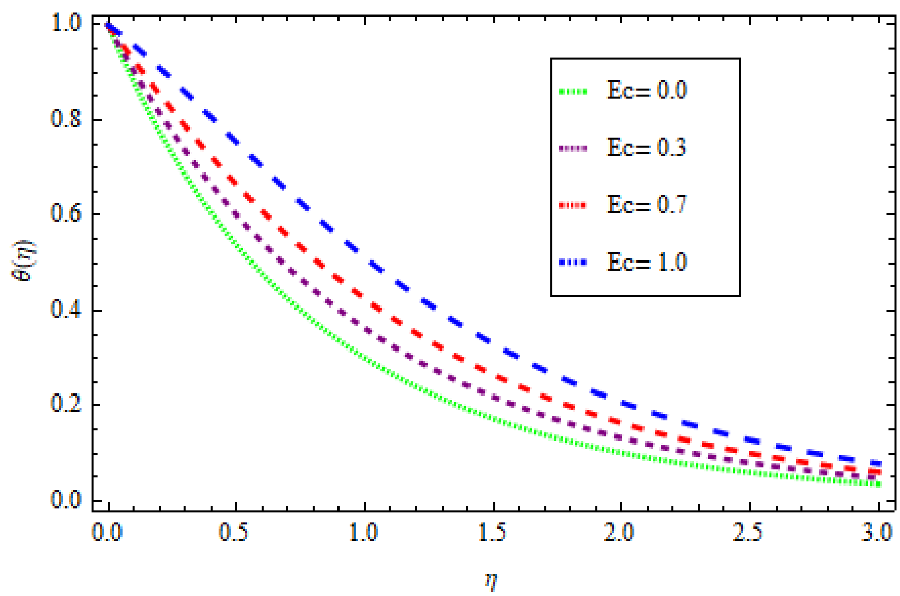

Figure 7. The influence of the temperature profile for different values of E_c, when $K = 0.1,\ fw = 0.3,\ \lambda = 0.2,\ \Pr = 5.0,\ \beta = 0.4,\ a^* = b^* = 0.2.$

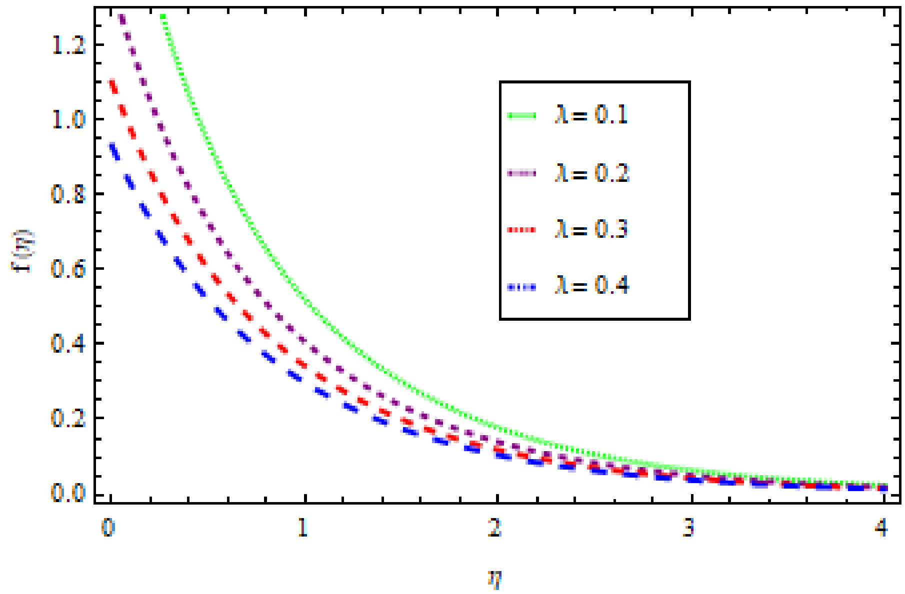

Figure 8. The influence of the velocity profile for different values of λ, when $K = 0.1,\ fw = 0.3,\ \beta = 0.2,\ \Pr = 5.0,\ Ec = 0.4,\ a^{*} = b^{*} = 0.2.$

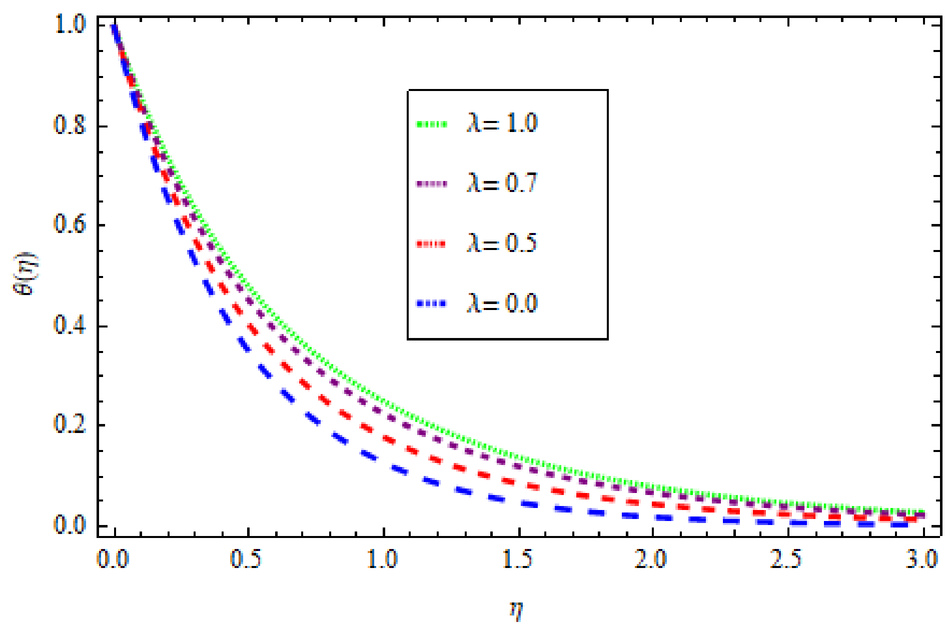

Figure 9. The influence of the temperature profile for different values of λ, when $K = 0.1,\ fw = 0.3,\ \beta = 0.2,\ \Pr = 5.0,\ Ec = 0.4,\ a^{*} = b^{*} = 0.2.$

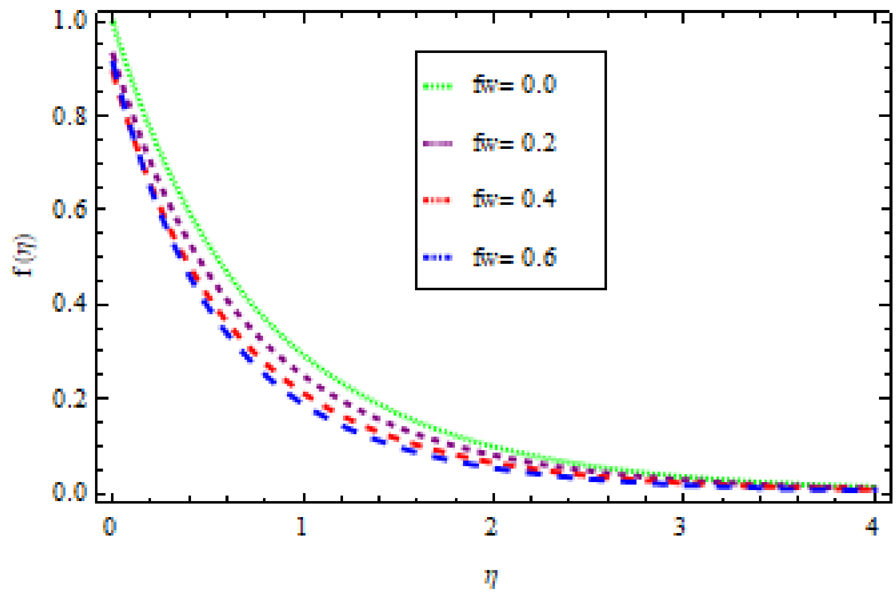

Figure 10. The influence of the velocity profile for different values of f_w, $K = 0.1, \beta = 0.4, \ \Pr = 5.0, \ Ec = 0.4, \ \lambda = 0.2, \ a^* = b^* = 0.2.$

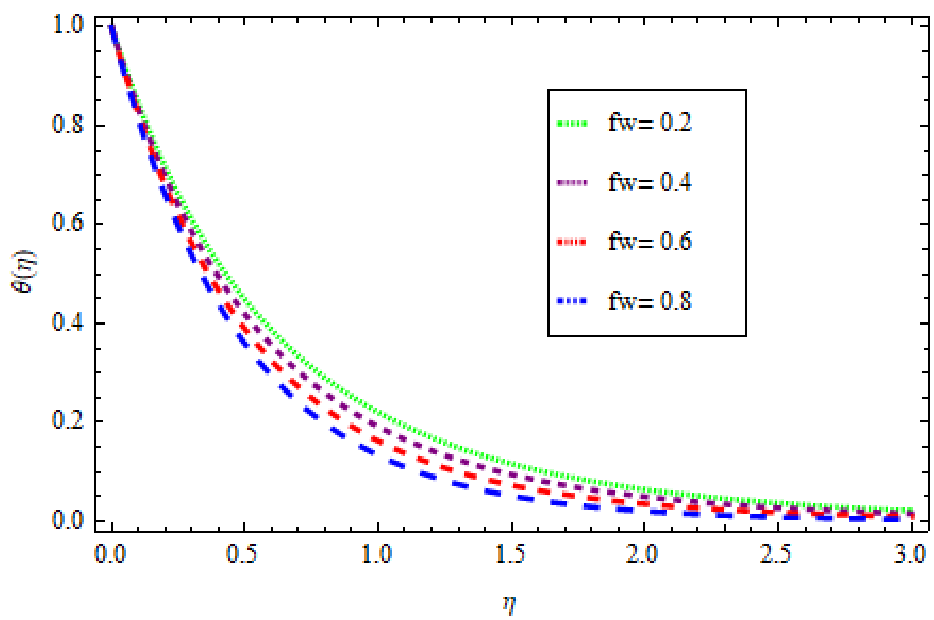

Figure 11. The influence of the temperature profile for different values of f_w, when $K = 0.1, \beta = 0.4, \ \Pr = 5.0, \ Ec = 0.4, \ \lambda = 0.2, \ a^* = b^* = 0.2.$

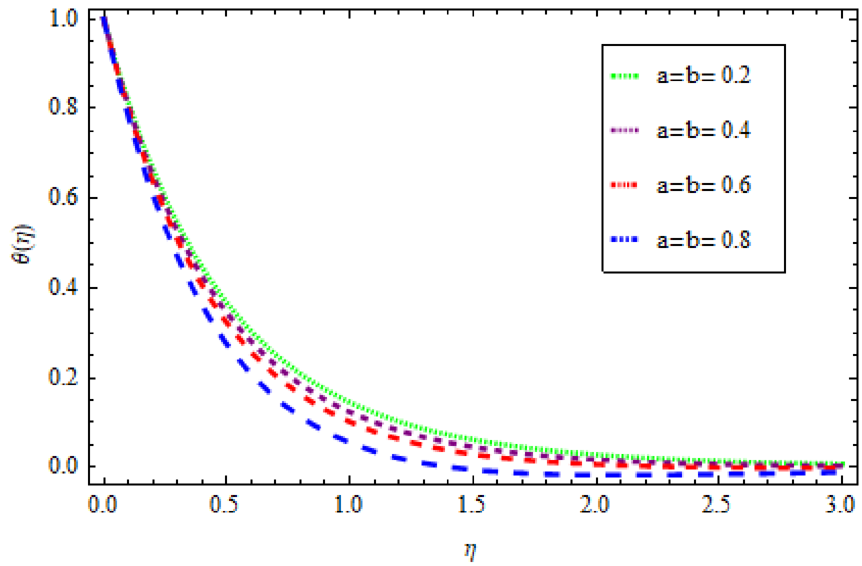

Figure 12. The influence of the temperature profile for different values of a^*, b^*, when $K = 0.1,\ \beta = 0.4,\ \Pr = 5.0,\ Ec = 0.4,\ \lambda = 0.2,\ f_w = 0.3.$

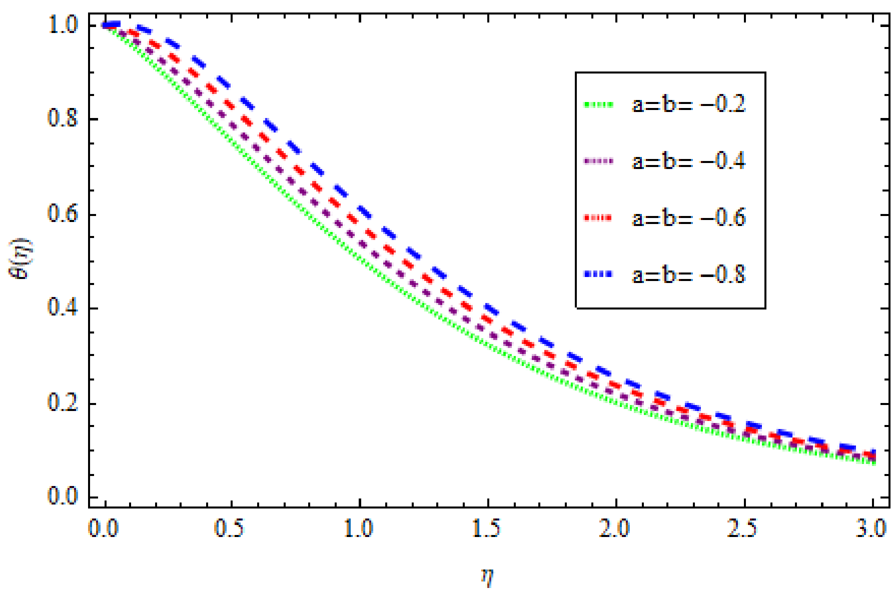

Figure 13. The influence of the temperature profile for different values of a^*, b^*, $K = 0.1,\ \beta = 0.4,\ \Pr = 5.0,\ Ec = 0.4,\ \lambda = 0.2,\ f_w = 0.3.$

5. Conclusions

The homotopy analysis aethod is a semi numerical scheme applied for the solution of the proposed model problem of heat transfer phenomena in viscoelastic fluid through a stretching sheet surface embedded in a porous medium with viscous dissipation of internal heat generation/absorption and slip velocity. Convergence analysis of the method is presented graphically. The effects of emerging parameters on the solution have been discussed in detail. It is observed that the suction parameter reduces the thickness of the boundary-layer flow. Similarly, the porosity and slip parameters have same effect on thickness of the boundary layer flow as observed in suction parameter. Also, the thermal boundary layer and temperature distribution increases with the increasing values of Eckert number. Additionally, the present work is compared with the published work reported by Rajagopal et al. [22] for limiting cases and good agreement is found.

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Nomenclature

x, y	Velocity components	κ	Thermal conductivity of the fluid	E_c	Eckert Number
ρ	Fluid density	T	Fluid temperature	λ	Velocity slip parameter
μ	Fluid viscosity	K	Porous Parameter	f_w	Suction velocity
A, r	Constants	q'''	Internal heat generation	C_f	Skin friction
μ_e	Dynamics viscosity	c_p	Specific heat	Nu	Nusselt number
T_w	Sheet temperature	a	Velocity Slip factor	R_{ex}	Reynolds number
T_∞	Ambient temperature	η	Similarity variable	β	Porous parameter
v_w	Suction velocity	υ	Kinematics viscosity	a^*, b^*	Heat generation parameters
u	Sheet velocity	$\Pr$	Prandtl number		

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