SUMMER SCHOOL

NEW PERSPECTIVES ON CONVEX GEOMETRY

Castro Urdiales, September 3rd-7th, 2018

Organized by:

María A. Hernández Cifre (Universidad de Murcia)

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SCHEDULE

Monday September 3rd

8:30-8:50 Registration

8:50-9:00 Opening

9:00-10:30 Rolf Schneider: *Convex Cones, part I* (3h course)

10:30-11:00 Coffee break

11:00-12:30 Manuel Ritoré: *Isoperimetric inequalities in convex sets, part I* (3h course)

12:45-13:15 Jesús Yepes: *On Brunn-Minkowski inequalities in product metric measure spaces*

13:15-15:30 Lunch

15:30-16:20 Franz Schuster: *Spherical centroid bodies*

16:30-17:20 Daniel Hug: *Some asymptotic results for spherical random tessellations*

Tuesday September 4th

9:00-10:30 Monika Ludwig: *Valuations on convex bodies, part I* (3h course)

10:30-11:00 Coffee Break

11:00-11:50 Andrea Colesanti: *Valuations on spaces of functions*

12:00-12:45 Rolf Schneider: *Convex Cones, part II* (3h course)

13:00-13:30 Thomas Wannerer: *Angular curvature measures*

13:30-15:30 Lunch

15:30-16:15 Manuel Ritoré: *Isoperimetric inequalities in convex sets, part II* (3h course)

16:30-17:20 Vitali Milman: *Reciprocal convex bodies, the notion of "indicatrix" and doubly convex bodies*

Wednesday September 5th

9:00-10:30 Richard Gardner: *Structural theory of addition and symmetrization in convex geometry, part I* (3h course)

10:30-11:00 Coffee Break

11:00-12:30 Gil Solanes: *Integral geometry and valuations, part I* (3h course)

12:45-13:30 Monika Ludwig: *Valuations on convex bodies, part II* (3h course)

13:15-15:30 Lunch

FREE AFTERNOON

Thursday September 6th

9:00-10:30 Apostolos Giannopoulos: *Banach-Mazur distance to the cube, part I* (3h course)

10:30-11:00 Coffee Break

11:00-12:30 Francisco Santos: *Hollow lattice polytopes and convex geometry, part I* (3h course)

12:45-13:15 Dmitry Faifman: *Extensions and non-extensions of the Weyl principle*

13:15-15:30 Lunch

15:30-16:15 *Richard Gardner: Structural theory of addition and symmetrization in convex geometry, part II* (3h course)

16:30-17:20 Dmitry Ryabogin: *On a local version of the fifth Busemann-Petty problem*

Friday September 7st

9:00-9:45 Apostolos Giannopoulos: *Banach-Mazur distance to the cube, part II* (3h course)

10:00-10:50 Artem Zvavitch: *Bezout inequality for mixed volumes*

10:50-11:10 Coffee Break

11:10-11:40 David Alonso: *The negative square correlation property on ℓ_p^n-balls*

11:50-12:35 Gil Solanes: *Integral geometry and valuations, part II* (3h course)

12:45-13:30 Francisco Santos: *Hollow lattice polytopes and convex geometry, part II* (3h course)

13:00-15:30 Lunch

FREE AFTERNOON

ABSTRACTS OF COURSES

Structural theory of addition and symmetrization in convex geometry

Richard Gardner

The idea of combining two compact convex sets K and L in $\mathbb{R}^n$ to make a new one is fundamental in convex geometry. Such operations are usually called additions, the best known being Minkowski addition, which yields the vector sum $K + L$ of K and L. It has been said that the Brunn-Minkowski theory itself arises from combining two notions, Minkowski addition and volume. Other very useful additions include L_p addition, Orlicz addition, and M-addition. There are also dual versions of these additions that combine two star-shaped sets to make a new one. Understanding the relationship between these additions, their properties, and their characterization via these properties, has been the focus of recent research initiated with Daniel Hug and Wolfgang Weil of the Karlsruhe Institute of Technology and continued with other colleagues such as Lukas Parapatits, Franz Schuster, and Deping Ye.

The idea of symmetrization—taking a subset of Euclidean space (for example) and replacing it by one which preserves some quantitative aspect of the set but which is symmetric in some sense—is both prevalent and important in mathematics. The most famous example is Steiner symmetrization, introduced by Jakob Steiner around 1838 in his attempt to prove the isoperimetric inequality. Steiner symmetrization is still a very widely used tool in geometry, but it and other types of symmetrization are of vital significance in analysis, PDE's, and mathematical physics as well. An ongoing joint project with Gabriele Bianchi and Paolo Gronchi of the University of Florence focuses on symmetrization processes that associate to a given set in $\mathbb{R}^n$ one that is symmetric with respect to a subspace. Such processes include Schwarz, Minkowski, Minkowski-Blaschke, central, and fiber symmetrization. Again, a structural theory emerges, allowing enhanced understanding of the relationship between these symmetrizations, their properties, and their characterization via these properties.

These talks will address both additions and symmetrizations. In fact, the two are linked, because an addition often generates a symmetrization. For example, the Minkowski symmetral of a compact convex set with respect to a subspace is, up to a constant factor, the Minkowski sum of the set and its reflection in the subspace. While explaining the current state of knowledge, we will highlight open problems whose solutions would advance the field.

Western Washington University, USA

Banach-Mazur distance to the cube

Apostolos Giannopoulos

A question of particular geometric interest is the study of the radius of the Banach-Mazur compactum with respect to ℓ_∞^n, defined by

$$\mathcal{R}_\infty^n = \max\{d(X, \ell_\infty^n) : \dim X = n\}.$$

This is the smallest positive constant α with the property that for every symmetric convex body K in $\mathbb{R}^n$ we may find a symmetric parallelepiped P such that $P \subseteq K \subseteq \alpha P$. The question to determine the asymptotic behavior of the sequence $\mathcal{R}_\infty^n$ as n tends to infinity was posed by Pełczynski. Bourgain and Szarek showed that $\mathcal{R}_\infty^n = o(n)$, and later Szarek and Talagrand obtained the estimate $\mathcal{R}_\infty^n \leqslant cn^{7/8}$. The best known upper bound, since 1993, states that there exists an absolute constant $c > 0$ such that

$$\mathcal{R}_\infty^n \leqslant cn^{5/6}$$

for any $n \geqslant 1$. A second proof of the same upper bound, based on spectral sparsification, was given a few years ago by P. Youssef. We present both arguments and we discuss the connection of the problem with the property of restricted invertibility of matrices and the proportional Dvoretzky-Rogers factorization.

In the other direction Szarek, using random spaces of Gluskin type, proved that $\mathcal{R}_\infty^n \geqslant c\sqrt{n}\log n$. This means that $\mathcal{R}_\infty^n$ has order of growth strictly greater than $\sqrt{n}$; in other words, ℓ_∞^n is not an asymptotic center of the Banach-Mazur compactum. Very recenty, K. Tikhomirov showed that there exists a symmetric convex body G_m, which is a random polytope in $\mathbb{R}^n$ with $2m := 2n^{c_1}$ vertices, such that $d(B_1^n, G_m) \geqslant n^{5/9}\log^{-c_2} n$, where B_1^n is the n-dimensional cross-polytope, and hence $R_\infty^n \gg n^{5/9}$. We present the main ideas behind this improvement.

University of Athens, Greece

Valuations on Convex Bodies

Monika Ludwig

A fundamental theorem of Hadwiger classifies all rigid-motion invariant and continuous valuations on convex bodies, that is, functionals on compact convex sets in $\mathbb{R}^n$ that satisfy the inclusion-exclusion principle,

$$\mathrm{Z}(K) + \mathrm{Z}(L) = \mathrm{Z}(K \cup L) + \mathrm{Z}(K \cap L)$$

for K and L such that $K \cup L$ is convex. Under weak additional assumptions, a valuation is a finitely additive measure and hence Hadwiger's theorem is a counterpart to the classification of Haar measures.

Hadwiger's theorem characterizes the most important functionals within Euclidean geometry, the $n+1$ intrinsic volumes, which include volume, surface area, and the Euler characteristic. In recent years, numerous further functions and operators defined on various spaces of convex bodies were characterized by their properties.

An overview of these results and an outline of some of the proofs will be given. In particular, the following will be discussed:

(i) Real valuations
(ii) Vector and tensor valuations
(iii) Convex body and star body valued valuation

The focus is on valuations that intertwine the $\mathrm{SL}(n)$.

Technische Universität Wien, Austria

Isoperimetric inequalities in convex sets

Manuel Ritoré

We consider the partitioning problem for a Euclidean convex body: how to separate it into two regions of prescribed volumes minimizing the perimeter of the interface. We first consider basic aspects of the problem such as existence and regularity of minimizers. Then we shall focus on its isoperimetric profile, that should be understood as an optimal isoperimetric inequality in the convex body, and consider its behavior under Hausdorff convergence, as well as its concavity properties and their consequences.

Universidad de Granada, Spain

Hollow lattice polytopes and convex geometry

Francisco Santos
(joint work with Mónica Blanco and Oscar Iglesias)

We will study enumeration of lattice polytopes, mainly focussing on the methods arising from Convex Geometry. Thus, we will use the technique of the covering minima and the successive minima in order to obtain good bounds for the volume of the above mentioned polytopes.

Universidad de Cantabria, Spain

Integral geometry and valuations

Gil Solanes

A classical task in integral geometry is to compute the average of a certain geometric quantity over all positions of an object. A fundamental example is Blaschke's principal kinematic formula which gives the measure of positions in which a convex body A (moving under the Euclidean group) intersects a fixed convex body B. The result is expressed in terms of the so-called intrinsic volumes of A and B. Similar results exist for A, B non-convex, and also in other spaces such as the sphere. Kinematic formulas in Euclidean space, and also in the sphere, have important applications in different areas of mathematics, and also in other fields such as medicine or material science.

From a broader perspective, integral geometry deals with natural operations on the space of valuations. A valuation is a finitely additive functional on the space of convex bodies (or another class of nicely enough subsets). Since Hadwiger's classification of continuous invariant valuations in Euclidean space, the theory of valuations has played a fundamental role in convex and integral geometry, with essential contributions by McMullen, Schneider, Klain and others. More recently, Alesker has discovered several natural structures in various spaces of valuations [1, 2]. These structures are closely related to classical constructions of integral geometry. For instance, Alesker's product of valuations is dual to kinematic formulas, in a sense made precise by Fu and Bernig [3].

The goal of the course will be to present these recently discovered structures on the space of valuations and show their connections to integral geometry. In particular, we will see how difficult problems in integral geometry, such as the determination of kinematic formulas in hermitian spaces, have been solved thanks to the use of these structures.

References

[1] S. Alesker, Description of translation invariant valuations on convex sets with solution of P. McMullen's conjecture, *Geom. Funct. Anal.* **11** (2001), 244–272.

[2] S. Alesker, Hard Lefschetz theorem for valuations, complex integral geometry, and unitarily invariant valuations, *J. Diff. Geom.* **63** (2003), 63–95.

[3] A. Bernig, J. H. G. Fu, Hermitian integral geometry. *Ann. of Math.* **173** (2011) ,907–945.

[4] A. Bernig, J. H. G. Fu, G. Solanes, Integral geometry of complex space forms, *Geom. funct. Anal.* **24** (2014), n.2, 403–492.

Universitat Autònoma de Barcelona, Spain

Convex cones

Rolf Schneider

Recent work by several authors (McCoy, Tropp, Amelunxen, Lotz) on the use of convex optimization for signal demixing (and other problems) has brought into focus the following geometric question. Let C, D be closed convex cones (with apex o) in $\mathbb{R}^d$. Suppose that C is fixed and D undergoes a uniform random rotation ρ. What is the probability that C and ρD have a non-trivial intersection (that is, $C \cap \rho D \neq \{o\}$)? The answer is immediate if C and D are linear subspaces: if the sum of their dimensions is at most d, the probability of non-trivial intersection is 0, otherwise it is 1. For general convex cones, spherical (or conical) integral geometry provides an explicit answer, in terms of the conical intrinsic volumes of C and D. Unfortunately, the latter cannot be computed in general. The work of the above-mentioned authors has revealed that for convex cones there is a concentration result, approximately imitating the behavior of subspaces. With a convex cone, one can associate a 'statistical dimension', and the conic intrinsic volumes concentrate around this number (in a precise way). This implies a phase transition: if the sum of the statistical dimensions of C and D is considerably less than d, then the probability in question is close to 0; if that sum is considerably larger than d, then the probability is close to 1 (there are explicit estimates). The purpose of my Mini Course is to present this new development from the point of view of convex geometry.

Albert-Ludwigs-Universität Freiburg, Germany

ABSTRACTS OF INVITED LECTURES

The negative square correlation property on ℓ_p^n-balls

David Alonso-Gutiérrez

A log-concave random vector X in $\mathbb{R}^n$ is said to verify the negative square correlation property with respect to an orthonormal basis $\{\eta_i\}_{i=1}^n$ if for every $i \neq j$

$$\mathbb{E}\langle X, \eta_i\rangle^2\langle X, \eta_j\rangle^2 - \mathbb{E}\langle X, \eta_i\rangle^2\mathbb{E}\langle X, \eta_j\rangle^2 \leq 0.$$

If X is uniformly distributed on an ℓ_p^n-ball, this property is verified with respect to the canonical basis, implying the variance conjecture on this family of convex bodies. In the case of $p = 2, \infty$, the negative square correlation property is true not only with respect to the canonical basis but with respect to any orthonormal basis. We will study this property with respect to any orthonormal basis for other values of p, showing that the ℓ_p^n-balls satisfy this property with respect to any orthonormal basis if and only if $p \geq 2$.

Universidad de Zaragoza, Spain

Valuations on spaces of functions

Andrea Colesanti

Valuations on function spaces is a rather recent area of research, which has been rapidly growing in the last years, mainly inspired by the rich theory of valuations on convex bodies. A valuation on a space of function X is a mapping from X to the reals, which is additive with respect to the operation of maximum and minimum of functions (which, roughly speaking, replace union and intersection involved in the definition of valuation on a family of sets). The aim of the talk is to describe the state of the art in this area, and present the main examples of functions spaces which have been studied, and the corresponding characterisation results that have achieved for valuations on these spaces.

Università degli Studi di Firenze, Italy

Extensions and non-extensions of the Weyl principle

Dmitry Faifman
(joint work with Andreas Bernig, Gil Solanes and Thomas Wannerer)

A famous theorem of H. Weyl, the tube formula, describes the volume of an epsilon-tube of a Riemannian manifold embedded in Euclidean space as a polynomial in epsilon, whose coefficients, remarkably, do not depend on the embedding. Through works of S.-S. Chern, H. Federer and others, those quantities generalize to curvature measures on Riemannian manifolds. Very recently, J.H.G. Fu and T. Wannerer characterized those curvature measures through universality to embedding. I will discuss extensions, non-extensions and analogies of some of those results to pseudo-Riemannian, Finsler, and contact manifolds.

University of Toronto, Canada

Some asymptotic results for spherical random tessellations

Daniel Hug
(joint work with Andreas Reichenbacher and Christoph Thäle)

In Euclidean space, the asymptotic shape of large cells in various types of Poisson driven random tessellations has been the subject of a famous conjecture due to David Kendall. Since shape is a geometric concept and large cells are identified by means of geometric size functionals, the resolution of the conjecture is connected with geometric inequalities of isoperimetric type and related stability results. The current work [1] explores specific and typical cells of random tessellations in spherical space. We obtain probabilistic deviation inequalities and asymptotic distributions for quite general size functionals. In contrast to the Euclidean setting, where the asymptotic regime concerns large size, in the spherical framework the asymptotic analysis is concerned with high intensities.

In addition to results for Poisson great hypersphere and Poisson Voronoi tessellations in spherical space, we report on the recent work [2] on splitting tessellation processes in spherical space, which correspond to STIT-tessellation models (<u>st</u>able under <u>it</u>eration) in Euclidean space and which have been studied intensively in recent years. Expectations and variances of spherical curvature measures induced by splitting tessellation processes are studied by means of auxiliary martingales and tools from spherical integral geometry. The spherical pair-correlation function of the $(d-1)$-dimensional Hausdorff measure is computed explicitly and compared to its analogue for Poisson great hypersphere tessellations. Various other cell characteristics can be treated as well and can be related to distributions of Poisson great hypersphere tessellations.

Karlsruher Institut für Technologie, Germany

Reciprocal convex bodies, the notion of "indicatrix" and doubly convex bodies

Vitali Milman
(joint work with Liran Rotem)

University of Tel Aviv, Israel

On a local version of the fifth Busemann-Petty problem

Dmitry Ryabogin
(joint work with Maria Angeles Alfonseca, Fedor Nazarov and Vlad Yaskin)

Let K be an origin-symmetric convex body in $\mathbb{R}^n$, $n \geq 3$, satisfying the following condition: there exists a constant c such that for all directions ξ in $\mathbb{R}^n$,

$$h_K(\xi)\,\mathrm{vol}_{n-1}(K \cap \xi^{\perp}) = c.$$

(here $\xi^{\perp}$ stands for a subspace of $\mathbb{R}^n$ of co-dimension 1 orthogonal to a given direction ξ, and $h_K(\xi)$ is the support function of K in this direction). The fifth Busemann-Petty problem asks if K must be an ellipsoid. We give an affirmative answer to this question for origin-symmetric convex bodies that are sufficiently close to an Euclidean ball in the Banach-Mazur distance.

Kent State University, USA

Spherical centroid bodies

Franz Schuster
(joint work with F. Besau, T. Hack, and P. Pivovarov)

Going back to C. Dupin and W. Blaschke, the notion of Euclidean centroid bodies, along with their associated isoperimetric inequalities by H. Busemann and C.M. Petty, forms a classical part of the theory of convex bodies. In this talk, we present a new definition of centroid bodies in spherical space, explore their basic properties, and discuss isoperimetric problems associated with them.

Technische Universität Wien, Austria

Angular curvature measures

Thomas Wannerer

The curvature measures of H. Federer introduced in his seminal work on the curvature of non-smooth subsets of $\mathbb{R}^n$ take on a particularly simple form in the special case of convex polytopes:

$$\Phi_k(P,U) = \sum_F \gamma(F,P)\mathrm{vol}_k(F \cap U),$$

where $P \subset \mathbb{R}^n$ is a polytope, $0 \leq k \leq n$ is an integer, $U \subset \mathbb{R}^n$ is a Borel subset, the sum extends over all k-faces of P, and $\gamma(F,P)$ is the external angle of P at the face F. Given any function f on the Grassmannian of k-dimensional linear subspaces of $\mathbb{R}^n$, we consider the weighted sums

$$\Phi(P,U) = \sum_F f(\overline{F})\gamma(F,P)\mathrm{vol}_k(F \cap U), \tag{1}$$

where the sum is over all k-faces of P and $\overline{F}$ is the translate of the affine hull of F containing the origin. The obvious question arises whether such expressions can be extended to curvature measures of more general subsets of $\mathbb{R}^n$; any linear combination of such curvature measures is called angular.

In this talk I will present a complete characterization of those functions f for which (1) extends to an angular smooth curvature measure on $\mathbb{R}^n$. Within the framework of S. Alesker's theory of valuations on manifolds the notion of angular curvature measure admits a natural extension to Riemannian manifolds. I will discuss how the aforementioned characterization of angular curvature measures can be used to prove a conjecture of A. Bernig, J.H.G. Fu, and G. Solanes on angular curvature measures in a Riemannian manifold.

Friedrich-Schiller-Universität Jena, Germany

On Brunn-Minkowski inequalities in product metric measure spaces

Jesús Yepes Nicolás
(joint work with Manuel Ritoré)

The well-known Brunn-Minkowski theorem says that

$$\mathrm{vol}\big((1-\lambda)A+\lambda B\big)^{1/n} \geq (1-\lambda)\mathrm{vol}(A)^{1/n}+\lambda\mathrm{vol}(B)^{1/n}$$

for any A, B non-empty (Lebesgue) measurable subsets of $\mathbb{R}^n$ such that their linear combination $(1-\lambda)A+\lambda B=\{(1-\lambda)a+\lambda b:\ a\in A,\ b\in B\}$, $\lambda\in(0,1)$, is also measurable.

Although it would be not possible to collect here all the generalizations of this inequality during the last decades, in this talk we will briefly comment some of them and we will discuss how it is possible to obtain some other Brunn-Minkowski type inequalities in a different setting.

In this regard, given one metric measure space X satisfying a linear Brunn-Minkowski inequality, and a second one Y satisfying a Brunn-Minkowski inequality with exponent $p\geq -1$, we will show that the product $X\times Y$ with the standard product distance and measure satisfies a Brunn-Minkowski inequality of order $1/(1+p^{-1})$ under mild conditions on the measures and the assumption that the distances are strictly intrinsic. The same result holds when we consider restricted classes of sets.

In particular, we will show that the classical Brunn-Minkowski inequality holds for any pair of weakly unconditional sets in $\mathbb{R}^n$ (i.e., those containing the projection of every point in the set onto every coordinate subspace) when we consider the standard distance and the product measure of n one-dimensional real measures with positively decreasing densities. This will allow us to get an improvement of the class of sets satisfying the Gaussian Brunn-Minkowski inequality.

Universidad de Murcia, Spain

Bezout Inequality for Mixed volumes

Artem Zvavitch

In this talk we will discuss the following analog of Bezout inequality for mixed volumes:

$$V(P_1,\dots,P_r,\Delta^{n-r})V_n(\Delta)^{r-1}\leq\prod_{i=1}^{r}V(P_i,\Delta^{n-1})\ \text{ for } 2\leq r\leq n.$$

We will briefly explain the connection of the above inequality to the original Bezout inequality and show that the inequality is true when Δ is an n-dimensional simplex and $P_1, \ldots, P_r$ are convex bodies in $\mathbb{R}^n$. We will present a conjecture that if the above inequality is true for all convex bodies $P_1, \ldots, P_r$, then Δ must be an n-dimensional simplex. We will show that the conjecture is true in many special cases and give actual examples of Bezout type inequality characterizing the simplex.

Finally, we connect the inequality to an inequality on the volume of orthogonal projections of convex bodies as well as present an isomorphic version of the inequality.

Kent State University, USA

ABSTRACTS OF POSTERS

Isodiametric and isominwidth inequalities for bisections of a planar convex body

ANTONIO CAÑETE
(joint work with Bernardo González)

Given a planar convex body K, a *bisection* B of K is a decomposition of K into two connected subsets K_1, K_2 by means of a simple curve. In this setting, we can define the *maximum relative diameter* of B by

$$d_M(P, K) = \max\{D(K_1), D(K_2)\},$$

where D is the usual Euclidean diameter, and we can consider the minimum value of this functional among all the bisections of K, namely

$$D_B(K) = \min_P d_M(P, K).$$

In the same spirit as the classical isodiametric inequality (Bieberbach, 1915), we obtain the isodiametric inequality for bisections

$$\frac{A(K)}{D_B(K)^2} \leq 2\,(\arctan(2) - \arctan(1/2)),$$

where $A(K)$ denotes the area of K, characterizing also the convex set providing the equality.

In a similar way, for a given bisection B of K, we consider the *maximum relative width* of B,

$$w_M(B) = \max\{w(K_1), w(K_2)\},$$

where w is the minimal width functional, and the corresponding minimum value

$$w_B(K) = \min_P w_M(K).$$

In this case, and analogously to Pál (1921), we study the relation between this functional and the area of the set, obtaining that

$$\frac{A(K)}{w_B(K)^2} \geq \frac{4}{\sqrt{3}},$$

with equality if and only if K is an equilateral triangle. Finally, we also study the corresponding reverse inequalities, following the ideas of Behrend (1937), focusing on the affine class of each set.

Universidad de Sevilla, Spain

Relating Brunn-Minkowski and Rogers-Shephard inequalities using the asymmetry measure of Minkowski

Katherina von Dichter
(joint work with René Brandenberg and Bernardo González Merino)

In this work we propose to improve Brunn-Minkowski and Rogers-Shephard inequality in terms of the asymmetry measure of Minkowski. We do a first step by computing some bounds via stability results of those inequalities.

Universidad de Murcia, Spain

Uniqueness of the measurement function in Crofton's formula with lines

Rikke Eriksen
(joint work with Markus Kiderlen)

For a convex body $K \subseteq \mathbb{R}^n$, Crofton's intersection formula states that the $(n-j)$th intrinsic volume of K can be written as a invariant integral of the $(k-j)$th intrinsic volume of the section $K \cap E$, where E is a k-plane, $0 \leq j \leq k < n$.

Motivated by results in stereology, we ask if there are other functionals ϕ, replacing the $(k-j)$th intrinsic volume, with this property. The answer is positve even when assuming that the functionals are continuous (w.r.t. Hausdorff metric), translation invariant valuations and we will give explicit examples. On the other hand, assuming in addition that ϕ is rotation invariant we get uniqueness, due to Hadwiger's theorem. Strengthening this, we show that the assumption of motion invariance is sufficient for uniqueness when $k = 1$, i.e. when intersecting with lines.

Furthermore we are able to characterize all functionals ϕ, as above, when assuming instead that the functionals are local, i.e. translation invariant, locally determined functionals [1], and considering intersections with lines.

References

1. W. Weil, Integral geometry of translation invariant functionals, II: The case of general convex bodies. *Integral geometry of translation invariant functionals, II: The case of general convex bodies.* Advances in Applied Mathematics 66 (2015) 46 − 79.

Aarhus University, Denmark

A discrete Borell-Brascamp-Lieb inequality

David Iglesias
(joint work with Jesús Yepes Nicolás)

Borell-Brascamp-Lieb's inequality states that for non-negative measurable functions f, g, h on $\mathbb{R}^n$ and $p > -1/n$, if they satisfy that $h(x+y) \geq (f(x)^p + g(y)^p)^{1/p}$ for every $x, y \in \mathbb{R}^n$ such that $f(x)g(y) > 0$, then

$$\int_{z\in\mathbb{R}^n} h(z)dz \geq \left(\left(\int_{x\in\mathbb{R}^n} f(x)dx\right)^q + \left(\int_{y\in\mathbb{R}^n} g(y)dy\right)^q\right)^{1/q}$$

where q satisfies $1/q - 1/p = n$.

This is a very important integral inequality by his own, but also implies many other famous inequalities, like *Prékopa-Leindler's inequality* or even the general *Brunn-Minkowski inequality*.

In this poster we present a discrete analog of the *Borell-Brascamp-Lieb inequality*. More precisely, we show that if $A, B \subset \mathbb{R}^n$ are finite sets, and f, g, h are non-negative functions on $\mathbb{R}^n$ satisfying $h(x+y) \geq (f(x)^p + g(y)^p)^{1/p}$, $p > -1/n$, for all $x \in A, y \in B$ such that $f(x)g(y) > 0$, then

$$\sum_{z\in A+B} h(z) \geq \left(\left(\sum_{x\in r_f(A)} f(x)\right)^q + \left(\sum_{y\in B} g(y)\right)^q\right)^{1/q}$$

where q satisfies $1/q - 1/p = n$ and the set $r_f(A)$ is a reduction of the set A which is obtained by substracting some points, by means of a recursive procedure depending on the function f.

Universidad de Murcia, Spain

Sharp Sobolev Inequalities via Projection Averages

Philipp Kniefacz
(joint work with Franz Schuster)

In this talk we present a family of sharp Sobolev-type inequalities obtained from averages of the length of i-dimensional projections of the gradient of a function. This family has both the classical Sobolev inequality (for $i = n$) and the affine Sobolev-Zhang inequality (for $i = 1$) as special cases as well as a recently obtained Sobolev inequality of Haberl and Schuster (for $i = n - 1$). Moreover, we identify the strongest member in our family of analytic inequalities which turns out to be the only affine invariant one among them.

Technische Universität Wien, Austria

Octonion-valued forms: cobblestones on the road to $Val^{Spin(9)}$

Jan Kotrbatý
(joint work with Thomas Wannerer)

In spite of being real, the canonical $U(n)$-invariant K'ahler 2-form is usually expressed in terms of complex-valued (coordinate) 1-forms. We generalize this formalism and consider the (no longer associative) algebra of forms with values in the octonions. In the poster, two main results based on this generalization are presented. First, a new explicit algebraic formula for the canonical $Spin(9)$-invariant 8-form, the Kahler form's 'octonionic sibling', is given in terms of the two octonionic coordinate 1-forms on the octonionic plane. Second, the $Spin(9)$- and translation-invariant differential forms on the sphere bundle of the octonionic plane as well as their exterior differentials are described in terms of certain octonion-valued forms too. This is an essential step towards the description of the algebra of $Spin(9)$- and translation-invariant continuous valuations on convex bodies in the octonionic plane.

Friedrich-Schiller-Universität Jena, Germany

Fractional Sobolev norms and BV functions on manifolds

Andreas Kreuml
(joint work with Olaf Mordhorst)

We extend the notions of fractional Sobolev seminorms and fractional perimeters to compact Riemannian manifolds. The dependence of both of these functionals on a parameter $0 < s < 1$ raises the question of convergence in the limit cases. For $s \to 1$, their asymptotic behaviour can be modeled by a larger class of non-linear integral operators, whose kernels concentrate on one point in the limit. Using a suitable covering of the manifold allows us to establish convergence of the functionals in question to $W^{1,p}$- and BV-seminorms which generalizes results by Bourgain, Brezis & Mironescu, and Dávila. In particular, the limit of fractional perimeters yields the perimeter functional which generalizes the notion of surface area to a broad class of sets.

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Functional Orlicz Affine and Geominimal Surface Areas

NICO LOMBARDI
(joint work with Deping Ye)

Affine and Geominimal Surface Areas have been developed during the years for convex bodies, concerning affine inequalities (i.e. isoperimetric or Blaschke-Santaló) and differential geometry. Many generalizations have been studied, like L^p and Orlicz versions of those again for convex bodies, and also, later, their functional counterpart.

We are going to present the Orlicz Geominimal surface area extension to convex functions, defined in a variational sense in [1].

Let $\psi\colon \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ be a convex function with $\mathrm{int}(\mathrm{dom}(\psi)) \neq \varnothing$, we denote by

$$X_\psi = \{x \in \mathbb{R}^n |\ \exists\ \nabla^2\psi(x) \text{ and it is invertible}\}.$$

Let $F_1, F_2\colon \mathbb{R}^n \to (0, +\infty)$ be two smooth enough functions. We require also some integrability conditions, for instance

$$0 < \int_{X_\psi} F_1(\psi(x))dx < +\infty,\ 0 < \int_{X_{\psi^*}} F_2(\psi^*(x))dx < +\infty,$$

where ψ^* is the Legendre transform of ψ, to guarantee that the following expressions are well-defined.

Let $h\colon (0, +\infty) \to (0, +\infty)$ continuous and $g\colon X_{\psi^*} \to \mathbb{R}_+$ measurable. Then the Orlicz mixed integral of ψ and g w.r.t. F_1, F_2 is defined as

$$V_{h,F_1,F_2}(\psi, g) = \int_{X_\psi} h\Big(\frac{g(\nabla\psi(x))}{F_2((x, \nabla\psi(x)) - \psi(x))}\Big)F_1(\psi(x))dx,$$

and then the geominal surface area is

$$G_{h,F_1,F_2}(\psi) = \inf\{V_{h,F_1,F_2}(\psi, g)|\ g \in L_{\psi^*},\ I(g, \psi^*) = (\sqrt{2\pi})^n\},$$

where L_{ψ^*} is the set of all log-concave functions defined on $X_{\psi*}$ and

$$I(g, \psi^*) = \int_{X_{\psi^*}} \mathring{g}(x)dx,$$

with $\mathring{g}$ the polar function of g.

The main idea of the work is to prove first that the infimum in the geominimal definition is actually a minimum. Then we want to extend this definition, and the Orlicz affine surface area, to log-concave and quasi-concave functions, studying also their valuation properties.

References

[1] U. Caglar, D. Ye, Affine isoperimetric inequalities in the functional Orlicz-Brunn-Minkowski theory, Advances in Applied Mathematics 81 (2016), 78-114.

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Cone valuations, Gram's relation, and flag-angles

Sebastian Manecke
(joint work with Spencer Backman and Raman Sanyal)

We study linear relations of interior and exterior angle sums. In both cases we prove that only one linear relation exists, one being Gram's relation. For this we generalize the usual notion of an angle with simple cone valuations and show these relations in this setup. The uniqueness follows from a connection between angle sums and the combinatorics of zonotopes. Surprisingly, angle-sums of zonotopes are independent of the notion of angle used.

We further introduce flag-angles, an analogue of flag-f-vectors and show that flag-angle sums again exhibit a connection to the combinatorics of zonotopes. This allows us to show that no further relation for flag-angles exist by proving that the flag-f-vector of the lattice of flats of zonotopes has no non-trivial linear relation.

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Valuations on Lipschitz functions: a characterization result

Daniele Pagnini
(joint work with Andrea Colesanti, Pedro Tradacete and Ignacio Villanueva)

A *valuation* on a lattice of functions $(X, \vee, \wedge)$ is a functional $\mu : X \to \mathbb{R}$ such that

$$\mu(u \vee v) + \mu(u \wedge v) = \mu(u) + \mu(v),$$

for every $u, v \in X$. Valuations on different function spaces have been studied, leading to many characterization theorems. These results provide integral representation formulas for valuations satisfying certain hypothesis such as continuity and invariance under some kind of transformation.

We hereby present a characterization result concerning valuations on the space $\mathrm{Lip}(S^{n-1})$ of Lipschitz continuous functions on the $n-$dimensional sphere, giving an integral representation formula for valuations which are continuous, rotation invariant and dot product invariant (i.e. invariant under the addition of linear functions).

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Rogers-Shephard type inequalities for finite measures in the Euclidean Space

Michael Roysdon

(joint work with David Alonso-Gutiérrez, María Hernández Cifre, Jesús Yepes Nicolás and Artem Zvavitch)

A central inequality to the theory of convex bodies is the Brunn-Minkowski inequality which states that, for any convex bodies $A, B \subset \mathbb{R}^n$, one has $|A+B|^{1/n} \geq |A|^{1/n} + |B|^{1/n}$, where $|\cdot|$ denotes the n-dimensional Lebesgue measure. In the 1950's Rogers and Shephard proved a sort of converse to this inequality which states that, for any convex body K, one has $|K+(-K)| \leq \binom{2n}{n}|K|$ with equality if, and only if, K is a simplex. In a joint work with David Alonso-Gutiérrez, María Hernández Cifre, Jesus Yepes Nicolás and Artem Zvavitch, we present an analogue of this inequality in the setting of general measures with certain properties. Another inequality of Rogers and Shephard is that which gives a lower bound of the volume of a K in terms of its maximal section and projection onto a linear subspace. A functional analogue of this will be presented in the case of finite Borel measure with quasi-concave densities and when K is selected to satisfy certain conditions.

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Volume inequalities for down-bodies

Shay Sadovsky

(joint work with Shiri Artstein-Avidan and Raman Sanyal)

We prove several sharp results for certain measures of symmetry, and also some new results of Mahler type, for a class of bodies called “down bodies”, also known as “anti-blocking bodies”.

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